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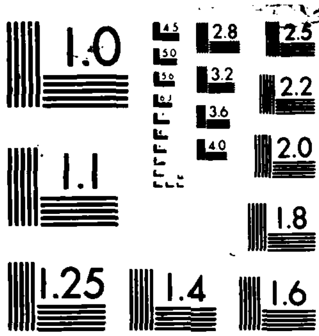

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FINAL REPORT

Investigation Into the Use of Normal and Half-Normal Plots for Interpreting Results from Screening Experiments

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Daniel, in 1959, proposed the use of half-normal probability plots as a means of interpreting the results from unreplicated 2^f or $2^{(f-p)}$ factorial experiments. Zahn, in 1975, suggested modifications to Daniel's approach and investigated some operating characteristics of these plots, along with techniques for estimating population sigma from the slope of the standardized contrasts plotted on half-normal probability paper. That work is reviewed here.

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19. Abstract (cont'd)

This project examines some properties and the usefulness of both normal and half-normal probability plots when applied to the interpretation of experimental data from screening experiments. The purpose of a screening experiment is to select from a very large number of potentially critical factors the ones that do have an important effect on the performance of a particular task, so that these may be investigated in depth.

Characteristics of the results from Monte Carlo simulations of 2^f experiments, where f = 5, 6, and 7, are studied. Artificial data from 5000 runs are created using as an error base a normal population with a mean of zero and a variance of one, to which from one to as many as 16 real effects are added of varying sizes. Ordered contrasts for both normal and half-normal plots are examined along with the "mislocation" of real effects at ranks other than where they are originally located prior to being combined with the error component. The relationship between number and sizes of real effects to percentage of real effects mislocated is determined.

The relative effectiveness of normal and half-normal plots is examined. The advantages and disadvantages of each are discussed in the context of the screening experiment. Examples are given of individual normal plots to show the unreliability of this graphic technique to detect marginal effects. Guardrails are supplied for half-normal plots for 2^f experiments, where k = 5 and 6. These guardrails are the theoretical sizes that contrasts must be to be called "real" at some probability level. The pros and cons regarding the use of these plots are discussed, both with and without being supplemented with the guardrails to facilitate the detection of real effects.

An annotated bibliography is supplied of 19 published papers relevant to this project, i.e., papers that describe alternative uses for probability plots and/or that have information which might improve the conduct of screening experiments. In addition, a list of publications are given which include Daniel's and Zahn's original papers on half-normal plots in their references. Numerous computer programs used for this project are provided.

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SECTION I

INTRODUCTION

The purpose of this project was to investigate the operating characteristics of normal and half-normal plots and to evaluate their effectiveness when used to interpret data derived from screening experiments.

CONTEXT

Modern industrial and military equipments are frequently quite complex with interacting elements that may affect operator performance in unforeseeable ways. For several decades, efforts to optimize the design of such equipment have depended in part on the results from experiments in which representative operators are placed in representative situations and are required to perform representative tasks on the different equipment configurations in order to determine which configuration results in the best cost-effective performance.

While recognizing that a relatively large number of factors related to the operator and the environment, as well as the equipment itself, affect performance in any given task, investigators have carefully avoided studying more than a few equipment factors -- usually less than four -- at a time in any single experiment (Simon, 1976b). To cover a more expansive space, a series of few-factor experiments from different parts of the space are usually run. This spotted approach, however, quickly becomes economically unfeasible and never adequately covers the multifactor space of interest nor obtains sufficient information regarding factor interactions. Consequently, the information needed to properly design equipment in which 10 or 20 or more factors may have an impact on engineering decisions and operator performance is never obtained.

Simon (1973, 1977a, 1977b) proposed a holistic methodology whereby a great many factors could be investigated economically in a single human performance experiment. Its application to naval training-simulator equipment was

proposed (Simon, 1979) and successfully employed (Westra, Simon et al., 1981; Westra, 1982).

ECONOMY

This holistic approach makes many-factor experiments economically feasible by collecting the data sequentially in small blocks. Rather than attempt to do this with a mosaic of small experiments located at different parts of the experimental space, the holistic approach collects the data with minimum precision over the total space and later improves the precision where it is needed. Initially, all of the potentially critical factors are studied in a screening design capable of defining the simplest model of the experimental world. While not precise, this early overview will reveal which factors and what segments of the total space are critical, usually a small fraction of the original effort. This screening phase reduces the data needed to bring the precision of the information to the required level. By testing after each block and collecting no more data once the experimental results adequately model the real world, considerable savings can be realized since a second- or third-order model is usually enough to approximate for all practical purposes the performance on a great many human performance tasks.

The interpretation of the results from a screening experiment requires a delicate balance between two opposing goals affecting the conduct of the future research. On the one hand, the investigator wants to reduce the number of candidate factors to a small enough number to make the subsequent data collection feasible. On the other hand, he wants to be certain that all factors likely to be critical in the task under investigation will be identified. When one must categorize a factor as critical or not on the basis of the screening data, the difficult decisions rest with the effects of marginal size.

MULTIPLICITY

When a great many constrasts are being evaluated for significance at the same time -- 31 or 63 are the numbers frequently found in screening experiments -- some effects can appear several times larger than the average

even when no effect is real. For example, in an experiment with 31 contrasts, the size of the largest contrast can be 2.4 times larger than the average contrast purely by chance alone. Using the traditional 0.05 significance level in such an experiment would cause unreal effects to be judged real in over half of all experiments being done (Daniel, 1959, p. 312).

Most human performance investigators tend to overlook this problem of multiplicity when they use the conventional F-test of significance to evaluate the data from an analysis of variance. This neglect can lead to some serious misinterpretations unless the proper corrections are made in the allowable error rate of the individual comparisons as well as that of the overall family of contrasts.

UNREPLICATED EXPERIMENTS

As the number of factors to be studied increases, the amount of data that must be collected also increases. Large multifactor experiments are economically feasible only because the sequential application of fractional-factorial designs allows the size of the screening experiment to increase at a far slower rate than the number of factors being investigated.

However, even when fractional-factorials of the appropriate resolution are used, the amount of data which must be collected can still reach impractical limits. Logistical conditions as well as time limitations may be operating, for example, the availability of experimental subjects (operators) and of the equipment and supporting staff. For that reason, when 31 to 64 experimental conditions are involved, the traditional luxury of replicating the experiment may not be possible. Consequently, this unreplicated screening experiment will provide no direct estimate of the error variance to serve as the denominator of the F-test used to evaluate the significance of the experimental effects.

CONFOUNDING

In the unreplicated experiment, therefore, error variance traditionally is estimated by other methods, namely: (1) Obtaining it from the results of

similar experiments done at another time, and (2) Pooling the variance of higher-order interaction effects on the a priori assumption that they are not likely to be significant.

Neither technique is really justified in human performance research. For one thing, because of poor subject sampling procedures and of the large number of uncontrolled or unidentified factors that may differentially affect different experiments which may appear superficially the same, using the error variance from another experiment is risky business. For another, higher-order interactions -- the term usually refers to effects that include third-order or higher interactions -- may sometime be significant. This is particularly true in human performance experiments in which equipment and/or performance measurements are improperly scaled.

In screening experiments, the focus of this report, highly saturated fractional-factorials will be employed. Higher-order interaction effects, although expected to be insignificant, are generally confounded with the effects of interest and therefore unavailable for estimating error. While subsequent data collection will unconfound the critical effects, for the screening phase, some alternative way of estimating the error variance must be found when an unreplicated, fractional-factorial design is employed.

DATA ANOMALIES

When data are sparsely drawn from coordinates over a larger experimental space (the screening experiment), when the performance of human operators is involved, and when the tasks are performed on complex equipment which may occasionally break down, irregularities in the experimental data may be expected. Without a dedicated effort to do so, these may not be recognized when they occur nor ever discovered after the data has been fed into a computer and regurgitated as an F-table of an analysis of variance. Conclusions drawn from such distorted data therefore are likely to be erroneous unless the investigator finds ways of discovering the abnormalities.

HALF-NORMAL PLOTS

The problems cited above are not unique to screening experiments and must be attended to. In large multifactor screening studies, the sequential approach should handle any overconcern with the confounding of effects during the early stages of the investigation. But there remain the problems of data interpretation brought about by the absence of an external estimate of error, the confusion from multiplicity, and the distortion from outliers. Half-normal plots, when introduced, seemed to help reduce, if not solve, these problems. In addition, this graphic presentation provides the investigator with a quick overview of his results, facilitating interpretation beyond any statistics.

Daniel (1956, 1959) is most frequently credited as being the one who popularized the use of the half-normal plot to interpret the results from unreplicated 2^f and 2^{f-p} factorial experiments and to search for anomalies in the data. These balanced designs are the ones most commonly used in screening studies. An example of how the imaginary results from an experiment using a 2^4 design would appear on a half-normal plot is shown in Figure 1.

This half-normal plot shows the empirical cumulative distribution of the set of 15 orthogonal, unsigned contrasts when plotted onto a special grid with the contrasts plotted on the abcissa and the positions of the expected values of order statistics for a half-normal distribution for 15 cases plotted on the ordinate. If no real effects are present in this data, the plots differing solely by chance would tend to fall along a straight line. In our example, however, the five largest points deviate sufficiently from the line formed by the remaining points. One would conclude that those five effects are probably real and that the remaining points are made up of error contrasts.

Daniel (1959) proposed the use of standardized half-normal plots. The raw contrasts, x_i, are standardized, X_i, by dividing each by an initial estimate of the standard error of the contrasts. He then plotted "guardrails" on his grid, i.e., lines connecting critical values representing the amount a contrast that might exceed the central slope of the null experiment purely by chance by some probable amount.

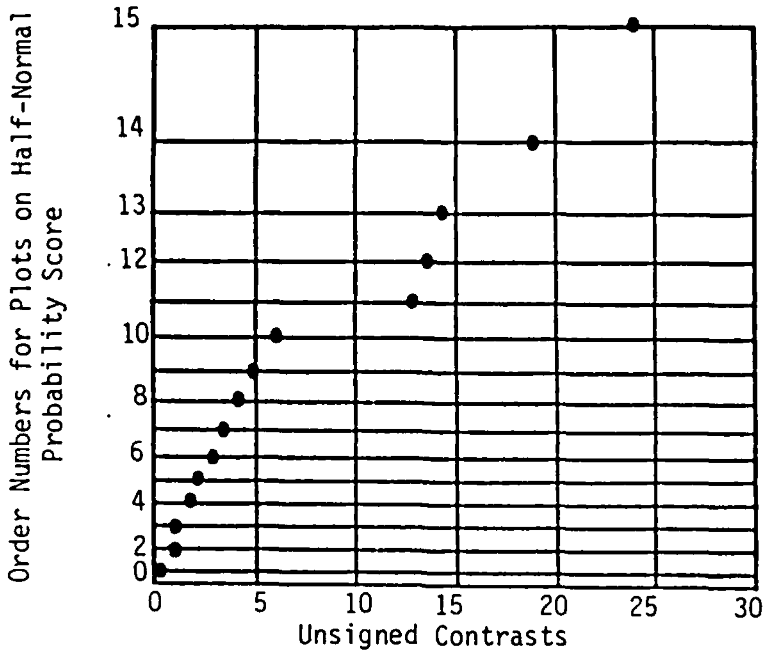

Figure 1. Example of Half-Normal Plot for a 2^4 Factorial Experiment

Daniel (1959) also showed how the ordered data of the half-normal plot might provide clues to abnormalities in the data, detecting such things as bad values, heteroscedasticity, and defective randomization. Later, Daniel (1976) suggested that the full-normal plot, in which the signs of the contrasts are retained, would provide more information about the data characteristics than the half-normal plot of the absolute contrasts.

Zahn (1975a, b) suggested changes in Daniel's plotting procedures. He also proposed to use the slope, SL, of the plotted error contrasts on this standardized grid to provide a final internal estimate the standard error, s_c, from which an estimate, s, of the population sigma can be derived. In a series of Monte Carlo studies, he compared several methods of estimating the slope.

SECTION II

REVIEW OF DANIEL'S AND ZAHN'S PAPERS

Daniel's and Zahn's papers on half-normal plots are reviewed here briefly as background for the present effort. While Hazen (1914) suggested the principle of linearizing the normal distribution in a study of floods, in recent years, Daniel's work is generally credited for popularizing and expanding the applications for normal and half-normal plots.

The extent to which people have found the half-normal plot to be useful is reflected in the number of papers between 1959 and the present that referenced Daniel's work (Appendix A). In the discussion below, Daniel's procedures are described as he did them, although subsequently Zahn suggested changes.

DANIEL'S PAPER

Daniel (1959) proposed the use of the half-normal plot as a graphic tool for evaluating and interpreting certain classes of experimental data. He noted that bad values, heteroscedasticity, dependence of variance on means, and some types of defective randomization have characteristic forms on the plots. He also suggested that the half-normal plot might be used in an unreplicated experiment to estimate the population σ, where σ^2 equals the variance of the data, and to use that information to identify which effects in a 2^f or 2^{f-p} experiment were significant.

Daniel modified normal probability paper to fit the half-normal case by deleting the given normal probability scale, P, and replacing it with one suitable for the half-normal case, where P' = 2P - 100. After marking off the ordinate scale with those revised values, he selected specific P' values as the plotting positions for k ranks using this equation:

$$P'_i = (i - 0.5) \div k;\ i = 1, 2, 3 \ldots k. \quad (1)$$

Daniel plotted the unsigned contrasts on the grid's abscissa.

Daniel noted that if the experiment were a null one -- that is, no factor had a real effect -- s_c can be roughly estimated by the contrast, x_a in k contrasts, where

$$a = \text{Integer nearest } x_{0.683k} \tag{2}$$

For k = 31, 63, and 127, the s_c would be the contrasts at ranks 22, 44, and 86 respectively on the ordered contrast scale. However, if some effects are real, they must be removed before Equation 2 would be used.

Daniel suggested that by standardizing the half-normal plot, fixed "guardrails" could be drawn at different α values, when α is the probability of calling an observed contrast real when in fact it is not. He standardized his contrasts, x_1, by dividing each one by the "best" estimate of the standard error of the contrasts, x_a, in a null experiment.

Daniel used as his test statistic (on which the location of a guardrail was based):

$$t_1^{\Delta} = x_{1,k} \div x_{a,k} \tag{3}$$

where:

$x_{1,k}$ is the largest in absolute magnitude of the k contrasts from a random normal variable with a population mean of zero and variance σ^2.

$x_{a,k}$ is the contrast at the rank determined by Equation 2, serving as an initial estimate of σ.

Daniel (and later Zahn) noted that the distribution of $\log_{10} t_k^{(0)}$ "is quite closely approximated by a normal distribution" (p. 319). When the t-statistic is plotted on a normal probability scale it appears as a straight line.

Daniel drew a guardrail, for example, to yield an error rate of α=0.05 false positives per experiment by drawing a line through the 0.05 critical values for the statistics t_{15}, t_{14}, t_{13}, and t_{12}.

Daniel proposed that guardrails representing a rather large error rate (i.e., false positive) be used, e.g., an α of 0.40 or even 0.80, "knowing that this will produce a number of 'false alarms'. The justification is of course in the increased sensitivity to small effects... Erroneously selected effects have good prospects of being exposed in later work. Missed real effects on the other hand are likely to be dropped from study, and if they are numerous, loss of knowledge of their combined effects may be serious." (Daniel, 1959, p. 322-325).

DANIEL'S CONCLUSIONS. Daniel concluded that "when only a small proportion of the totality of contrasts have effects, this plot can be used to make judgments about the reality of the largest effects found" (p. 339).

In general, effects or outliers have to be rather large, i.e., more than four times larger than x_a (the estimated σ) to be clearly visible. Single outliers may frequently be detected from the plots when the smaller values, presumably error contrasts, do not point toward the origin. Corrective techniques were suggested (p. 331).

Daniel showed how multiple plots can improve the interpretation of split plot designs (p. 329). This is accomplished by plotting separately the positive and negative halves of the effect used to block on, done of course on a grid suited to the smaller number of contrasts per plot. The contrast used in blocking is omitted. One might expect different error variances for the two plots, a fact that is hidden in the combined plot and one that could distort the interpretation of the results.

Daniel also examined how variations in the plot pattern might be used to warn of deviations from normality in the data (p. 331-338). However, he pointed out that certain unusual patterns might have multiple causes. No systematic investigation of this process was pursued in his paper.

It is only fair to add that Daniel recognized that his paper was not definitive and that a better understanding of how and what he had proposed was still needed. He also emphasized the subjective basis of this graphic technique and warned that using it in a routine way as a substitute for an anslysis of variance might be "catastrophic" (p. 338). He noted that on an individual basis, there can be wide variations in the appearance of the plots, even when no effects were introduced into the original data. He raised questions regarding the best method of estimating the standard error of the contrasts and wondered whether the 2^{p-q} experiment should be at least partially duplicated. He pointed out the importance of not relying solely on this graphic technique. He emphasized the dangers of a univariate (referring to the response) rather than a multivariate approach to research (p. 339).

ZAHN'S PAPERS

Zahn (1975a, b) wrote two papers on the construction and operating characteristics of the half-normal plot. In the first paper, he proposed a change in the way in which Daniel prepared the half-normal grid and exposed a major flaw in the way Daniel selected the critical values for his guardrails. In the second paper, Zahn evaluated several ways of estimating the error variance (based on the slope of standardized error contrasts) from the data of an unreplicated experiment.

Zahn (1975a) suggested reversing the scales that Daniel plotted on the ordinate and abcissa of the plotting grid, "to make the half-normal plot correspond more closely to the usual regression analysis graph on which the random variable is plotted as the ordinate" (p. 191). The half-normal percentiles were plotted on the horizontal axis. While configuring the grid either way would provide the same information, noticing which is being used before interpreting the plot patterns is important since real effects deviate from the null line in opposite directions in the two cases. Failing to perceive this difference could lead to serious errors in interpretation.

Zahn showed how Daniel erred in his calculations of the guardrails. The $t_i = x_i/x_a$, $i = 1, 2, \ldots, n$ which Daniel used are not half-normally distributed although he plotted them against percentiles of the standard half-normal distribution (p. 192). This discrepancy, however, proves to be small after the number of plotting positions exceed 20 (p. 193), the case of an experiment of size 2^{f-p}, where (f-p) equals five. On the standardized grid, drawing a line from the original through the value at the rank nearest to the 0.683 probability value will represent the expected values at all ranks in a null experiment.

Zahn noted a more serious error in the calculations Daniel used to plot the position of his guardrails and the error rate they are intended to control (p. 195). Daniel, in calculating his t-statistic, failed to change the sample size in x_a for each rank, which resulted in erroneous tables from which his critical values were selected.

The choice of critical values depends on what aspect of false positive behavior one is trying to control. Zahn wished to control the probability of a non-zero family error rate, the PER. This is the probability of obtaining at least one false positive in the family of contrasts when no real contrasts are present. In the null situation, each test assumes that no effects are present, and yields at least one false positive if and only if the largest contrast is declared significant. This occurs when the test statistic for examining the largest contrast is larger than its α level critical value. In that case, under the hypothesis that all contrasts equal zero, PER equals α (p. 195).

Zahn proposed a theorem with limited proof, namely, that the "half-normal plot using α-level critical values has PER equal to or less than α), regardless of how many real contrasts of various sizes are present in the experiment being analyzed (p. 195). The equation for the probability error rate for e independent tasks is:

$$PER = 1 = (1 - \alpha)^{e}, \tag{4}$$

This equation is not completely accurate when applied to ordered data, for in that case, the contrasts are not independent, although Equation 4 assumes they are.

In practice, however, it may be wise to set the guardrail with an α value equal to a PER for (e + 1) contrasts as calculated for the F-test. Zahn (p. 198) cites as an example a study in which an F-test was used to determine whether or not four effects were real (out of the nine being examined) at a 0.05 significance level. The F-test in that case had an approximate PER of 0.37. On a half-normal plot, the same effects were declared significant when the 0.40 guardrail was used (p. 198).

Zahn's guardrails differ from Daniel's because Daniel miscalculated his t-statistic, t_i. The significance of any contrast depends not only on its distance from the expected contrast but also its rank. Thus the t-statistic, to be calculated using Equation 3, requires that the numerator and denominator in the t-statistic be reduced one rank each time a contrast is declared significant in order to test the next lower contrast.

Because Zahn generated his error contrasts directly, when he refers to the size of a real effect in terms of σ -- a symbol used in this paper to represent the population σ -- he is using it to represent the standard error of contrasts. The reader should be aware that the standard error of contrasts equals the population sigma times the square root of four over n. This fact does not affect the comparisons he made in his study. It can matter, however, when we wish to put Zahn's results in a broader context and compare them with the results in the investigation for this project. Similarly, the slopes of his error contrasts actually approximate the standard error of the contrasts although he refers to them as σ. From this point, when we refer to Zahn's σ's, we shall put them in quotation marks, thus "σ", to remind us that it is really the standard error of the contrast.

Half-normal plots may be used to: (1) Detect real effects, and (2) Estimate the error variance. Zahn compared Daniel's procedure for using the half-normal plot, referred to as Version 0, against several versions of his

own. Whereas Daniel had used a single number to estimate "σ", i.e., the contrast falling at the rank nearest the 0.683 percentile of the ordered contrasts, Zahn, in order to obtain a more stable and accurate estimate, combined selected error contrasts to calculate the slope of the regression line of the plots on a standardized half-normal grid. This slope is an estimate of "σ."

ZAHN'S PROCEDURES FOR ESTIMATING σ. Zahn's Version X' procedure is this: After obtaining the ranked unsigned contrasts, he plots them at the appropriate plotting positions of a standardized half-normal grid. He still uses the rank nearest the 0.683 percentile as his initial estimate of "σ" to standardize the scale of his vertical axis. The critical values for the guardrails are determined and plotted. Then starting with the largest contrast, he determines whether it is plotted beyond the guardrail. If so, he declares it significant and looks at the next largest, and so forth until a contrast is plotted within the guardrail or "until x_{n-r} is encountered" (p. 202). The remaining, smaller contrasts are all declared insignificant and are called the error contrasts. He replots these error contrasts on a new half-normal grid against the appropriate expected values of normal order statistics, $z_{i,e}$ where i is the ith rank and e is the number of error contrasts. He next computes s_f, the final estimate of "σ", where s_f is the slope of the ordinary least squares regression line through the origin of x on z, fitted to the smallest $m = [0.7(e+1)]$ of the e error contrasts. The equation for this final estimate of "σ" is:

$$s_f = \sum_{i=1} x_i z_{ik} \Big/ \sum_{i=1}^{m} z_{ik}^2, \qquad (5)$$

where:

$m = [0.7\ (e + 1)]$.

The 0.7 value, the proportion of the error contrasts (plus one) that were used to estimate the slope, was empirically selected in Zahn's (1975b) second paper as being "a reasonable choice" (p. 206-208).

Zahn investigated several other versions which he felt might improve the detection process. In his Version S, instead of standardizing the contrasts with the rank nearest the 0.683 percentile as his initial estimate of "σ", he used the slope of the regression line calculated with the smallest standardized contrasts up to and including the contrast at the rank nearest to the 0.683 percentile.

In his Versions XR and SR, Zahn wanted to see if replotting and reassessing would improve the power of the half-normal plot. In Version XR, if the largest contrast were found to exceed the guardrail, the remaining ordered contrasts would be restandardized and replotted using as an estimate of "σ" the value at the rank nearest the 0.683 percentile for the remaining k-1 contrasts and replotting on a new scale for k-1 points. This process is iterated until no effects are found to exceed the guardrail. In Version SR, at each stage of the detection process, "σ" is estimated from the slope of [0.67 k'] against k' normal order statistical values, where k' is the number of contrasts being examined for significance, a number which reduces at each iteration. When no more significant effects are determined, then k' = e.

Zahn noted that all of the versions are based on the assumption that the investigator knows how many contrasts are real and how many are error (p. 203). He tried another version, R, in which he also revised the initial estimate of "σ" after the largest effect of any k' effects is declared significant and before the next ranked contrast is tested. In Version R, Zahn estimated "σ" from a slope based on half the number of the contrasts being examined. This allowed him, therefore, to include up to half of the original k contrasts, rather than 0.7 k, as in Version X.

Others have investigated different techniques for estimating σ in an unreplicated design. These include the efforts by Lloyd (1952) using generalized least squares instead of ordinary least squares, and by Wilk,

Gnanadesikan, and Freeny (1963) using a maximum likelihood estimate of σ^2. None seemed to justify their application.

ZAHN'S INVESTIGATIONS. Zahn (1975b) performed an empirical Monte Carlo study -- 1000 runs -- of half-normal plots limited to the k = 15 contrast case. He first generated a set of pseudo-normal standard normal deviates to represent the 15 error contrasts. He then compared the sensitivity of the five techniques -- versions -- in detecting real effects at different α settings when the number and size of the real contrasts were varied.

Whereas Daniel (1959) had investigated situations in which only one real contrast was present, ranging in size from one "σ" to six "σ", Zahn studied two situations: Type 1 Situation: r real contrasts, all of size d; Type 2 Situation: r real contrasts all of size d and r' real contrasts all of size d'. For the Type 1 study, r equalled 1, 2, 4, and 6 real contrasts out of 15 contrasts and d equalled from zero to 8 "σ." In the Type II situation, r and r' both equalled either one or two, and d equalled 2, 4, or 6 "σ" and d' equalled 4, 6, or 8 "σ." The sensitivity of each version was tested for α = 0.05, 0.20, and 0.40.

Zahn also compared the above versions with the _nomination_ approach. In that approach, the investigator decides _a priori_ to combine the higher order interactions to form an estimate of error variance (Pearce, 1953). As stated in the introduction to this paper, the assumption must be made that these higher order effects are negligible and that they are not confounded with lower-order effects, and if it not a valid assumption, the results are flawed.

Zahn used two criteria for evaluating a version's performance: (1) Detection rate, i.e., the proportion of real contrasts present in a situation which are detected, and (2) False positive behavior, the probability that at least one null contrast is declared significant, or the PER.

ZAHN'S CONCLUSIONS. It is difficult to draw sharp conclusions from Zahn's (1975b) results, although he does so to some extent within the limits of his investigation. Still it is apparent from the data that there are interactions

among many of the factors being manipulated. Results often depend on what values of n, r, and d are involved.

Some of his conclusions are:

1. Detection rates decrease as size of the real effects contrasts decrease.

2. Detection rates decrease as the number of real contrasts increases.

3. Size and number of real contrasts interact in their effect on the detection rates (p. 206).

4. Increasing α from 0.05 to 0.40 increases the detection rate but the probability of getting false positives also increases.

5. Though detection rate varied considerably from version to version, all false positive rates remain close to their respective α's. The largest differences occur when the one real contrast is small (p. 204).

6. Version R tended to be poorer than the other versions when the number of real effects were small (1 to 4 out of 15 contrasts), but unlike the other versions, it did not totally collapse when there were six real effects. However, its detection rate is still low and the estimate of "σ" is biased (p. 210). This could be explained by the fact that when there are 6 real contrasts out of only 15, the chances increase that the denominator of the test statistic would include the smaller real contrasts (p. 205).

7. Versions XR and SR yielded poorer detection rates when compared with Versions X and S respectively when two or four real effects were present in the 15 contrasts. In addition to being poorer, the iterative versions require more calculations (p. 205). Zahn concluded that revising the estimate of "σ" after each significant result did not improve the detection rate and is not recommended (p. 210).

8. Among all versions, Zahn recommended using Version S (p. 205), although when an inspection is made of his tables, it can be seen that in some cases this superiority is not large (particularly when compared with Version X), and in some cases, it may not exist at all.

9. Zahn recommended using the half-normal plot rather than the nomination approach if equivalent PERs are desired, unless almost all null contrasts can be accurately nominated (p. 210). For screening purposes, one cannot use the nomination approach because the higher-order interactions are confounded with lower-order effects so that both can bias one another. This confounding is temporarily acceptable in screening experiments, since with a sequential methodology, one expects to eventually discover and isolate these effects.

10. Overall, Zahn's results suggest that detection rates are relatively poor in situations of considerable practical interest, that is, when there are more than just a few real effects and when the effects were not too large. In Table 1, partial data from Zahn's Table 5 (1975b, p. 207) shows the detection rate, PER, and final estimate of "σ" for Version S with two or four real effects of size d = 2"σ", s"σ", or 6"σ" critical values of 0.05, 0.20, and 0.40.

TABLE 1. EMPIRICAL BEHAVIOR OF VERSION S IN 2- and 4-SITUATIONS WITH REAL CONTRASTS OF SIZES 2"σ", 4"σ", and 6"σ" PRESENT, USING α = 0.05, 0.20, and 0.40 (From Zahn's Table 5 [1975b, p. 207])

Number of Real Contrasts Present	Criterion	α	Size of the Real Contrasts Present 2"σ"	4"σ"	6"σ"
	Detection Rate	0.05	.080	.539	.954
		0.20	.219	.821	.997
		0.40	.365	.920	1.000
2	PER	0.05	.023	.044	.046
		0.20	.121	.166	.181
		0.40	.283	.341	.335
	Final Estimate of "σ"	0.05	1.148	1.089	1.004
		0.20	1.117	1.019	.982
		0.40	1.069	.971	.954
	Detection Rates	0.05	.037	.270	.864
		0.20	.134	.654	.988
		0.40	.247	.835	.999
4	PER	0.05	.008	.002	.000
		0.20	.050	.010	.000
		0.40	.143	.028	.000
	Final Estimate of "σ"	0.05	1.335	1.432	1.124
		0.20	1.297	1.235	1.002
		0.40	1.236	1.111	1.013

[True σ = 1]

SECTION III

RATIONALE FOR THE PRESENT PROJECT

Daniel and Zahn's work on half-normal plots became the starting point for the present project. There appeared to be a number of reasons to examine further the operating characteristics of both normal and half-normal plots and to evaluate how plots might be employed when applied to the unique features that characterize the screening experiment.

Specifically, the major motivation for the present project derived from the following beliefs and for the reasons given:

1. <u>Little work had been done to investigate the operating characteristics of half-normal plots since Zahn's work in 1975</u>.

Zahn (1977) was asked if he were aware of any additional investigations of the operating characteristics of the half-normal following his work. He answered that after 1975 he did not continue in that line of research but that he was not aware of any work that had been done.

In a cursory review of the literature in which Daniel's and Zahn's work are referred to (see Appendix A), it is interesting to note that out of the more than 100 papers listed in the citation indexes, Zahn's work is only referred to by others three times. In reports in which the half-normal plot was used to test significance, decisions were generally made by "eye-balling" the data rather than by using the more precise guardrails.

2. <u>Up to now, most of the work on plots as a test of significance had been with the half-normal plots, while the full-normal plot had been used to evaluate data quality</u>.

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After using the half-normal plot for some time, Daniel (1976) decided that a great many defects in experimental data "are strongly sign dependent, and

all are properties of subsets of the data set which are obscured in the half-normal plots by overaggregation" (p. 149). Daniel and others dropped the half-normal plot and began to use the full-normal primarily for evaluating data quality rather than as an aid to the detection process.

Draper and Smith (1981) described both the normal and half-normal plot as a means of evaluating residuals but without using guardrails to facilitate decisions regarding outliers.

3. Zahn's investigation was limited primarily to smaller designs than are likely to be used in screening experiments, suggesting that certain limitations noted by him might not apply with the larger designs.

The bulk of Zahn's (1975b) empirical study of the half-normal plot had been limited to the 2^f designs, where $k = 4$, and $n = 16$. When he concludes that the plots might be useful if the number of real effects were limited to four, it was because four contrasts represented one-fourth of his data and there would be only 11 contrasts with which to estimate the error variance. When he frequently detected less than 60% of the real effects which were two to four times the size of the standard error of the contrasts (which, for his work, translates into one and two times the size of his population σ), this too reflects the fact that with the smaller design, sensitivity will tend to be poorer.

It seemed desirable to determine whether the same problems would occur proportionately when the size of the experiment was increased. Thus, while four factors may decrease the sensitivity of a design with 15 contrasts, eight are unlikely to have as strong an effect on one with 31 contrasts, and the chances of picking out suitable null contrasts should increase. Furthermore, it seemed important to consider plot effectiveness when dealing more with real effects of marginal size.

4. Plot effectiveness would be better when used in screening designs where we are more concerned with avoiding Type II errors than avoiding Type I during the detection process.

In a screening experiment, the investigator is more concerned about rejecting potentially critical factors than he is about erroneously accepting some null ones. At this phase of the experimental program, his data is not so precise that he should dare to eliminate factors with marginal effects. As Daniel suggested, for screening we should select our critical values nearer the 0.40 α level than at the 0.05 level. A reexamination of the data with an emphasis on reducing Type II errors at the expense of making Type I errors seemed justified.

5. Plots frequently provide the data in a form that is easier to interpret than the conventional F-test and in some cases may be the only technique available.

There is no information inherent in the graphic approach that cannot be obtained from the data in numerical form. However, there is evidence to suggest that broad interpretations of the data can often be made more easily using the plots; on the other hand, precision is more readily obtained using numerical data. The visual presentation of plotted data, properly organized, can enable an investigator to see relationships and distortions that he might otherwise miss. Ordering by magnitude provides another overview of the data. It allows the investigator to quickly perceive groupings of factors -- i.e., main and interaction effects -- that have the greatest effect on performance. Such observations tend to favor the use of the plots in a screening study, where "ease of calculation is often more important than slight differences in stringency of conclusion" (Kurtz, Link et al., 1965). One big advantage of plots is that, without the benefit of statistics, they automatically adjust for multiplicity, which the (mis)user of the F-test may overlook when he tests a great many effects, one at a time, without adjusting for the fact that he would probably get some significant effects strictly by chance. With an unreplicated design, of course, no estimate of the error variance is available for doing the conventional F-test.

The above reasons supported the need for the present investigation of normal as well as half-normal plots on larger designs in order to maximize detection during the screening phase of an experimental program.

SECTION IV

CONSTRUCTING NORMAL AND HALF-NORMAL PLOTS

Normal and half-normal plots are graphic methods which allow the investigator to visually compare an empirically derived cumulative distribution of the effects with a cumulative distribution derived from a normally (or half-normally) distributed population. Once the contrasts have been calculated, the plots are constructed using normal probability paper.

One difference between the normal and the half-normal plot lies in the fact that the latter contains only the unsigned values of the k contrasts from a 2^{f-p} factorial experiment while the former retains the signs. Another difference is that the half-normal plot is essentially a folded normal plot, a fact that requires the plotting positions to change accordingly.

CONSTRUCTING NORMAL PLOTS

The normal plot for k contrasts from a 2^{f-p} fractional factorial experiment (where $k = n - 1 = 2^{[f-p]} - 1$) is constructed in the following manner on normal probability paper:

1. The k contrasts are plotted in order of magnitude, with signs intact, on the linear scale of a normal probability grid. Whether this scale is placed on the ordinate or the abscissa is somewhat arbitrary. Zahn (1975a) suggested that the contrasts should be plotted on the ordinate in order to make the half-normal grid correspond more closely to the usual regression analysis graph on which the random variable is plotted as the ordinate; this same argument would apply to the normal plot. Since investigators using these plots to present their experimental results have done it both ways, a reader should be careful to determine which is being used in order not to misinterpret the patterns of plotted points. In one case, real effects fall off below the zero line and in the other, above the zero line. The author saw one unsophisticated user become confused and misinterpret his own data,

claiming a number of effects to be real when, in fact, the plots fell on the wrong side of the line, a sign of truncated data, not significant contrasts.

2. The points are plotted against the percentiles of the standard normal distribution on the other axis. The particular plotting positions of the normal distribution can be obtained from a table of expected values of normal order statistics (Owen, 1962, for up to n = 50; Harter, 1961, for up to 99). If one needs expected values for sets larger than 99, tables of expected values of normal order statistics may be generated using a Monte Carlo simulation. The expected values of normal and half-normal order statistics are given for n = 32, 63, and 127 in Appendix B. The programs used to generate expected values of order statistics for this report are given in Appendix C (normal plots) and Appendix D (half-normal plots). Barnett (1976) discusses several methods of estimating these values, including an alternative, simple yet fully efficient, estimation method proposed by Gupta (1952).

An example of a normal plot is shown in Figure 2. Three effects -- two positive and one negative -- are presumably real because of their distance from the null line.

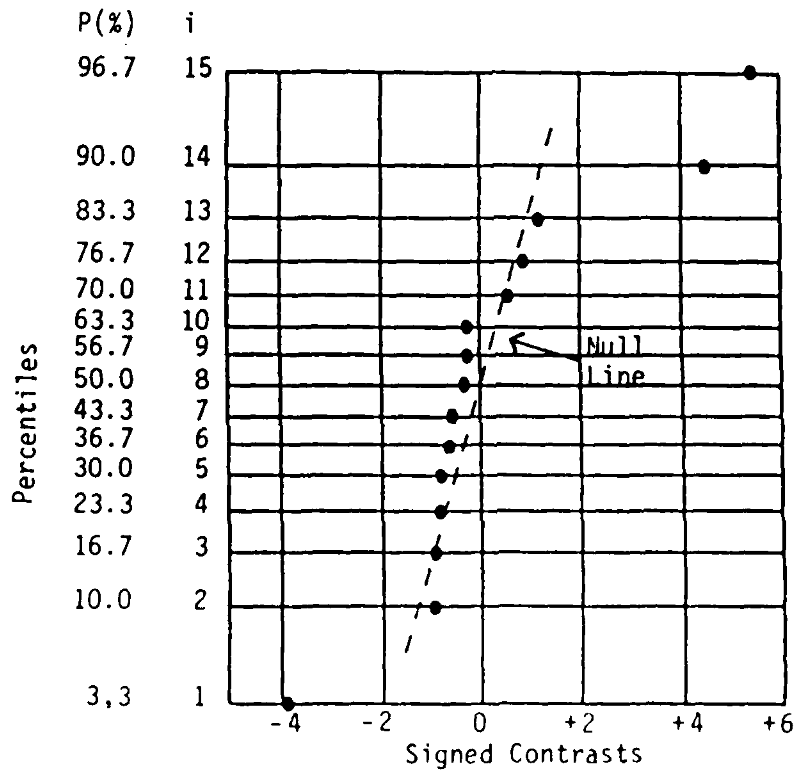


Figure 2. Example of Normal Plot

CONSTRUCTING HALF-NORMAL PLOTS

In the half-normal plot, the absolute contrasts -- values with the signs ignored -- are plotted against the percentiles of the standard half-normal distribution.

Percentiles for plotting positions of the half-normal distribution may be obtained from tables (Zahn, 1975a, between n = 6 and 20), simple calculations (Draper and Smith, 1966), or a computer program (Sparks, 1970; with a modification by Munford, 1972). The computer program used for this report is given in Appendix D. Leone, Nelson, and Nottingham (1961) also discuss some methods for estimating the expected values for the "folded normal distribution." Equation 1 may also be used to generate the expected values for the half-normal distribution, as Daniel did.

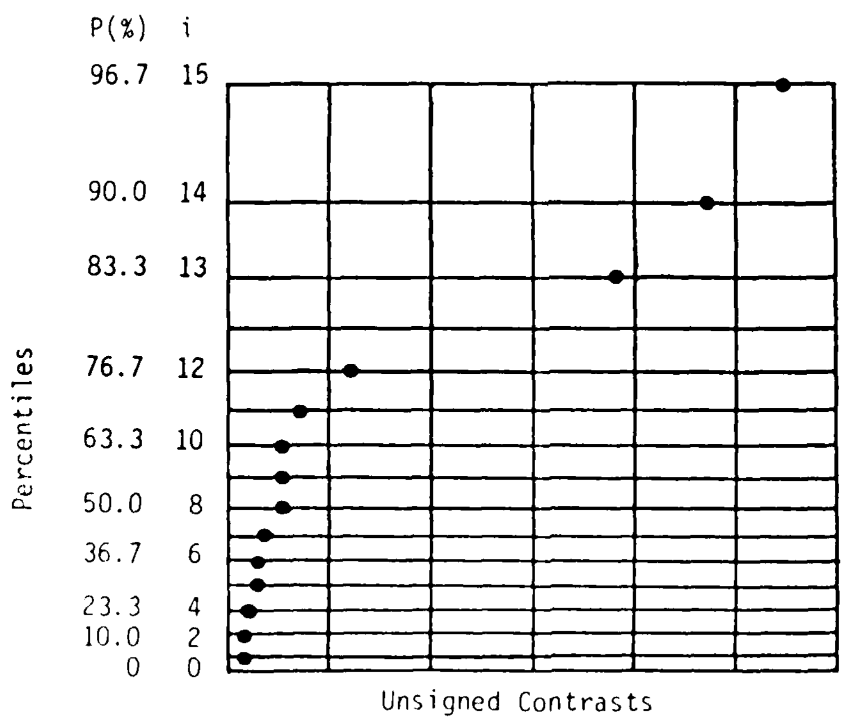


Figure 3. Example of Half-Normal Plot

(Using same data as in Figure 2)

The data used to generate the normal plot in Figure 2 was modified to generate an example of the half-normal plot shown in Figure 3.

STANDARDIZED PLOTS

Instead of the raw contrasts, the investigator may wish to plot the standardized contrasts on which the guardrails, proposed by Daniel, may also be plotted. In theory, these can be obtained by satisfying the following equation:

$$\text{Standardized contrast} = \frac{\text{Obtained contrast}}{\sigma\sqrt{4/n}} \qquad (6)$$

where σ is the population standard deviation and n is the number of conditions in the experiment from which the contrasts were generated. In practice, however, the population standard deviation is not known and an estimated value, s, is used instead. With unreplicated designs, as the review of Daniel's and Zahn's work revealed, it is not an easy nor obvious task to find an accurate s.

SECTION V

THE BASIC SIMULATION

For this project, the basic data for creating <u>normal</u> plots were generated using a Monte Carlo approach. Simulation efforts were carried out at two laboratories: the Human Factors Laboratory at the Naval Training Systems Center, Orlando, Florida, and the Statistics Department, Hollins College, Virginia.

While the half-normal plot will not be ignored, for this investigation the normal plot was emphasized, as stated in the section on "Rationale," because so much work had already been done with the half-normal and because Daniel had suggested more information is available using signed contrasts.

The primary variables in this simulation were: Size of the experimental design (n); number of contrasts (k); number of real effects (r); size of the real effects (d_r); and number of positive real effects (r+). The computer program used to generate the data base and perform the subsequent analyses is given in Appendix C.

CONSTRUCTING THE DATA BASE

The steps in the simulation process are:

1. Generate the sign matrix of a $2^f = n$ experimental design of desired size with the experimental conditions arranged in Yates' standard order.

2. Generate a set of n "scores" randomly selected from a normally distributed population with a mean score of zero and a variance of one ($\underline{N}$ [0,1]).

3. Randomly assign the numbers from Step 2 to the conditions in the experimental design from Step 1.

4. Calculate the $k = (n - 1)$ contrasts from the data in Step 3 using Yates' algorithm.

The above four steps produce a set of randomly distributed contrasts -- the mean differences between the two conditions of each experimental effect in the 2^f design -- from a null experiment. The standard error of the contrasts equals:

$$s_c = \sigma \sqrt{4/n} \tag{7}$$

where σ is the population σ, which is 1.00 in this simulation, and n is the number of independent data points in the experiment. In the unreplicated experiment, n equals the number of independent experimental conditions. In the replicated experiment, n equals the total number of conditions. The derivation of Equation 7 is given in Appendix E.

To introduce real effects into the data:

5. Create r real effects which may all be of the same or different magnitudes (d_r), of which r+ are positive and r- are negative.

6. The r effects are added to the signed contrasts which were randomly assigned (in Step 3) to the first r conditions of the design arranged in Yates' standard order. This assignment procedure for real effects is acceptable since no identification of specific effects is required.

7. The final contrasts, the composites of error and real effects, are ordered by the magnitude of the signed contrasts (when studying the normal-order plot) and of the unsigned contrasts (when studying the half-normal plots.

In the Monte Carlo program, the above steps were repeated 5000 times, the equivalent of running an experiment 5000 times. The mean and standard deviation of the contrasts occurring at each rank position of the ordered contrasts were obtained. Also, the real effects were tracked and the

number of times a real effect (regardless of magnitude) fell at each rank position was totalled.

BOUNDARIES OF THE EXPERIMENTAL SPACE EXAMINED IN THIS PROJECT

Since much of this effort was exploratory, a broad, multidimensional experimental space was examined unevenly, sampling relevant portions as questions arose.

Factor parameters included:

Size of experimental design:	$n = 32, 64, 128$
Number of ordered contrasts:	$k = 31, 63, 127$
Number of real effects	$r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 12, 16$
Number of positive effects	r_+ = The number ranged from 100% r down to 50% r in unit steps.
Size of effects:	$d = 0.50\sigma, 1.00\sigma, 1.15\sigma\ 1.25\sigma, 1.33\sigma, 1.50\sigma, 1.67\sigma, 2.00\sigma$ and occasionally intermediate values.

EXAMPLE

An example of the data from the above procedure for a normal plot is given in Table 2 for the situation:

n = Design size = Number of experimental conditions = $2^f = 32$
k = Number of effects (contrasts or mean differences) = $2^f - 1 = 31$
r = Number of real effects = 16 [Number of error contrasts, e = 15]
r+ = Number of positive real effects = 16
d_r = Magnitude of real effects = 1.00

TABLE 2. EXAMPLE OF ANALYSES: CONTRAST, NUMBER OF REAL EFFECTS, STANDARDIZED CONTRAST, AND CONTRAST STANDARD DEVIATION AT EACH RANK

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r=16+0-, d=1.00

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.612	1	-1.730	0.194
2	-0.441	4	-1.246	0.148
3	-0.335	5	-0.947	0.131
4	-0.252	16	-0.714	0.123
5	-0.180	34	-0.510	0.117
6	-0.117	43	-0.331	0.115
7	-0.058	75	-0.164	0.114
8	-0.003	111	-0.009	0.112
9	0.054	169	0.154	0.112
10	0.112	239	0.317	0.112
11	0.172	393	0.486	0.112
12	0.234	564	0.661	0.112
13	0.300	876	0.849	0.113
14	0.373	1334	1.054	0.115
15	0.452	2029	1.278	0.115
16	0.533	2958	1.507	0.116
17	0.611	3692	1.730	0.115
18	0.683	4092	1.931	0.114
19	0.748	4438	2.115	0.111
20	0.809	4641	2.288	0.110
21	0.865	4789	2.446	0.108
22	0.921	4816	2.606	0.109
23	0.973	4883	2.753	0.107
24	1.027	4936	2.905	0.109
25	1.081	4946	3.056	0.110
26	1.137	4961	3.216	0.113
27	1.198	4979	3.388	0.116
28	1.265	4985	3.577	0.121
29	1.346	4995	3.806	0.130
30	1.451	4996	4.103	0.146
31	1.622	5000	4.588	0.191

Three sets of data were obtained: (1) The mean contrasts at each position of the ordered data, $x_{i,k}$; (2) The standard deviation of those means at each position, $s_{i,k}$; and (3) The total number of real contrasts out of 5000 runs that appear at each rank of the ordered data, $\#R_{i,k}$.

The percentage of real effects that might occur at any rank in this simulation is:

$$\%R_{i,k} = [\#R_{i,k} \div 5000] \times 100. \tag{8}$$

We shall refer to the real contrasts that fall at a rank where only error contrasts were intended as "R-spillover." We shall refer to the error contrasts that fall at a rank where only the real effects were intended as E-spillover. The E-spillover is obtained for the appropriate ranks by subtracting the $\%R_{i,k}$ from 100% or the $\#R_{i,k}$ from 5000.

Standardized values of the mean contrasts, $z_{i,k}$, were also derived from the raw contrasts using the following equation:

$$z_{i,k} = x_{i,k} \div [\sigma \sqrt{4/n}] \tag{9}$$

where

$z_{i,k}$ = expected standardized contrast for rank i of k cases
$x_{i,k}$ = the expected contrast for rank i of k cases
σ = population σ
n = number of experimental conditions in the design

In this phase of the analysis, where results are aggregates of the results from 5000 runs, the correct population sigma, i.e., $\sigma = 1.00$, was used in Equation 9 as in Equation 7. In practice, and in the single experiment, it is unlikely that this value would be known and so would have to be estimated, s. We will also work with knowledge of how many real effects there are and at what ranks of the ordered contrasts they were intended to be located before being combined with the error component.

ACCURACY

While 5000 runs for the Monte Carlo studies seemed like a lot initially -- Zahn had used only 1000 runs in his study -- minor variations were observed when the same situations were recomputed or when computations were compared with other tables of expected values of normal order statistics or when the theoretical symmetry was not exact when expected at opposite ends of ordered data internal to certain tables. These variations were reduced when the Monte Carlo studies were made with 10,000 runs.

Careful study assured us that if the slight inaccuracy of the 5000 runs were accepted, there would be no alteration of any conclusion drawn in this report. By accepting the smaller number of runs, of course, computer costs and computing times were reduced considerably.

As a rule-of-thumb regarding the accuracy of our data, most of the time, contrasts and standardized contrasts were accurate to the second decimal place when rounded. As for the percent of real contrasts at each rank of the ordered contrasts, the numbers for the larger values seldom varied more than one or occasionally 2% from one set of data to another or within runs at each end of ordered contrasts which were theoretically the same. This latter inaccuracy is most visible in summary tables in which R-spillover values are given; this is because one set of data was used to prepare the tables while a second one, calculated with identical parameters, was used to provide clean copy for publication.

RELATING ZAHN'S PARAMETERS TO THOSE USED IN THE PRESENT INVESTIGATION

To relate certain conditions drawn by Zahn to those reached in the present investigation, it is necessary to use a common scale when referring to the size of an effect. When Zahn referred to a real contrast "of size 2σ," he meant that the effect was two times the size of the standard error of the contrasts (σ_c) in his experiment where n = 16. When we refer to a real contrast in this project "of size 2σ," we mean that the effect is two times the size of the population σ which is independent of the size of the

simulated data collection effort. This occurred because Zahn developed his 15 contrasts directly by sampling from a standard normal population of $\underline{N}[0,1]$, while in this project, the 16 "scores" for the null experiment were obtained from a standard normal population of $\underline{N}[0,1]$ and from those, the 15 contrasts were calculated. Zahn's population σ, to which he did not refer, was actually equal to 2.00, while ours was 1.00.

If one were only discussing the data within a study, which scale is used would be less important. If, however, we wish to relate the results from the two studies, the scales must be standardized.

Actually, when we express the size of a real effect in these simulations, three variables are involved, of which the third is determined by the other two. These variables are the population sigma, the size of the experiment, and the effect size. But effect size must be expressed in terms of population sigma or standard error of the contrast. Since the latter is a function of population sigma and experiment size, when contrast size is scaled by it, we cannot independently study the effects of real contrast size and experiment size. Because we ordinarily have no control over the population sigma in practice, it does us little good to be able to study independently the effect of real contrast size and changes in the population sigma.

For those reasons, we will scale the measures from both studies in terms of the population σ, since that is more in line with what would happen in the real world and because that will allow us to isolate the effects of real contrast size, d, and the experiment size, n.

One can relate the sizes of Zahn's effects to ours by converting them both to a multiple of population σ. Since he only used experiments of size $n = 16$, to express the size of his real effects in terms of the σ, as used in this project, it is only necessary to divide his values, which are in terms of s_c by two. In those terms, the effect of the population sigma remains constant, while the effects of contrast size and experiment size can be isolated.

SECTION VI

ANALYSES AND RESULTS

Simulated data for 2^f experimental designs were created and analyzed in order to:

1. Examine characteristics of expected values of normal order plots as a function of experiment size, n; number of ordered contrasts, k; number of real effects, r; size of real effects, d_r; and the number of positive real effects, r+.

2. Compare the relative effectiveness of normal and half-normal plots.

3. Examine individual plots.

4. Provide guardrails for evaluating half-normal plots for experiments of size, n = 32 and 64.

CHARACTERISTICS OF NORMAL PLOT DATA

The first analysis was carried out in order to examine the characteristics of expected values of ordered contrasts as a function of the size of the experiment, n; the number of ordered contrasts, k, which equal (n - 1) in an unreplicated design; the number of real effects, r; the number of positive real effects, r+; and the size(s) of the real effects (d_r).

Let us not be too concerned with quantification and pay more attention to the patterns formed by the data. For starters, let us look at the mean contrast data for these parameters:

$n = 32,\ k = 31,\ r = 8,\ d_s = +2\sigma$

The ordered mean contrasts, the number of times a real effect appears at each position (out of 5000 runs), the contrast standard deviations of the new contrasts at each rank, and the standardized mean contrasts are given in Table 3-a.

For this aggregate data from 5000 cases, the eight real contrasts are easily distinguishable from the error contrasts. The spillover of real effects into the ranks where only error contrasts are intended were trivial in this example. Only 0.3% of the real effects were located at the 23rd (error) rank out of 5000 runs.

Overall, the largest contrasts are the most variable. The contrast standard deviations at each rank show that variability is greatest at both ends of the intended error contrasts (ranks 1 and 23), being nearly double of what is found at the center of that set of 23 contrasts (rank 12). Unfortunately, these contrast standard deviations are not from normal distributions, particularly those of the larger and more important contrasts. Therefore, we cannot use them to establish the probability that a contrast at any particular rank would fall a certain distance from the expected value. Efforts to employ these contrast standard deviations to estimate the R-spillover at particular ranks proved too complex and inaccurate to pursue (see Appendix F).

Examination of the contrasts reveals an almost symmetrical set of values between ranks 1 and 23, with rank 12 at the pivot point. Since ranks 1 through 23 were intended to be the error contrasts in our simulation, and since there is practically no spillover of real effects into these ranks in this example, it is not surprising to find that these standardized "error" contrasts are almost identical -- within the two decimal point accuracy of our data -- to the expected values for normal order statistics (e.v.n.o.s.) for n = 23. These values are given in Table 4-a.

TABLE 3. EXAMPLE OF ORIGINAL AND STANDARDIZED ORDERED CONTRASTS, $\#R_{i,k}$, AND CONTRAST STANDARD DEVIATIONS

(a)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 8+0-, d=2.00

#	CONTRAST	# R		STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.681	0		-1.927	0.180
2	-0.523	0		-1.480	0.137
3	-0.431	0		-1.218	0.121
4	-0.360	0		-1.017	0.111
5	-0.301	0		-0.851	0.105
6	-0.250	0		-0.706	0.101
7	-0.202	0		-0.572	0.097
8	-0.159	0		-0.450	0.095
9	-0.118	0		-0.335	0.093
10	-0.078	0		-0.221	0.092
11	-0.039	0		-0.110	0.092
12	0.000	0	Intended Error	-0.001	0.091
13	0.038	0		0.106	0.091
14	0.077	0		0.217	0.091
15	0.116	0		0.329	0.093
16	0.158	0		0.446	0.094
17	0.201	0		0.569	0.096
18	0.248	0		0.702	0.100
19	0.299	0		0.845	0.104
20	0.359	0		1.014	0.111
21	0.430	0		1.217	0.121
22	0.525	3		1.485	0.139
23	0.681	20		1.925	0.182
24	1.495	4977		4.229	0.216
25	1.696	5000		4.798	0.175
26	1.831	5000		5.179	0.160
27	1.946	5000	Intended Real	5.505	0.154
28	2.053	5000		5.806	0.154
29	2.166	5000		6.125	0.160
30	2.300	5000		6.505	0.174
31	2.504	5000		7.081	0.217

(b)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 6+2-, d=2.00

#	CONTRAST	# R		STANDARDIZED CONTRAST	STD. DEVIATION
1	-2.198	5000		-6.216	0.292
2	-1.800	4991		-5.091	0.296
3	-0.680	8		-1.924	0.183
4	-0.521	1		-1.474	0.138
5	-0.428	0		-1.209	0.121
6	-0.357	0		-1.010	0.111
7	-0.297	0		-0.839	0.105
8	-0.247	0		-0.698	0.100
9	-0.200	0		-0.567	0.097
10	-0.157	0		-0.444	0.095
11	-0.116	0		-0.328	0.094
12	-0.076	0		-0.215	0.092
13	-0.037	0		-0.103	0.092
14	0.001	0		0.003	0.092
15	0.039	0		0.110	0.091
16	0.078	0	Intended Error	0.219	0.091
17	0.117	0		0.330	0.093
18	0.158	0		0.448	0.095
19	0.202	0		0.572	0.097
20	0.249	0		0.706	0.100
21	0.301	0		0.851	0.105
22	0.360	0		1.019	0.111
23	0.430	2		1.217	0.120
24	0.523	3		1.479	0.135
25	0.679	16		1.919	0.181
26	1.546	4980		4.374	0.227
27	1.771	4999		5.009	0.186
28	1.928	5000		5.452	0.176
29	2.073	5000		5.863	0.177
30	2.233	5000		6.316	0.186
31	2.452	5000		6.936	0.228

TABLE 4. EXPECTED VALUES OF NORMAL ORDER STATISTICS FOR n = 23 and n = 119.

(a) n = 23

#	ORDER STATISTIC
1	-1.92916
2	-1.48137
3	-1.21445
4	-1.01356
5	-0.84697
6	-0.70115
7	-0.56896
8	-0.44609
9	-0.32965
10	-0.21755
11	-0.10813
12	0.00000
13	0.10813
14	0.21755
15	0.32965
16	0.44609
17	0.56896
18	0.70115
19	0.84697
20	1.01356
21	1.21445
22	1.48137
23	1.92916

(b)

#	ORDER STATISTIC
1	-2.56913
2	-2.21696
3	-2.02022
4	-1.87972
5	-1.76869
6	-1.67595
7	-1.59577
8	-1.52463
9	-1.46051
10	-1.40183
11	-1.34775
12	-1.29728
13	-1.24987
14	-1.20522
15	-1.16277
16	-1.12237
17	-1.08368
18	-1.04661
19	-1.01088
20	-0.97638
21	-0.94304
22	-0.91077
23	-0.87936
24	-0.84880
25	-0.81900
26	-0.78996
27	-0.76152
28	-0.73371
29	-0.70645
30	-0.67970
31	-0.65344
32	-0.62763
33	-0.60223
34	-0.57720
35	-0.55252
36	-0.52820
37	-0.50418
38	-0.48042
39	-0.45697
40	-0.43374
41	-0.41075
42	-0.38800
43	-0.36541
44	-0.34301
45	-0.32079
46	-0.29874
47	-0.27682
48	-0.25504
49	-0.23338
50	-0.21183
51	-0.19037
52	-0.16900
53	-0.14771
54	-0.12649
55	-0.10532
56	-0.08420
57	-0.06312
58	-0.04206
59	-0.02103
60	0.00000
61	0.02103
62	0.04206
63	0.06312
64	0.08420
65	0.10532
66	0.12649
67	0.14771
68	0.16900
69	0.19037
70	0.21183
71	0.23339
72	0.25505
73	0.27683
74	0.29874
75	0.32080
76	0.34301
77	0.36541
78	0.38800
79	0.41075
80	0.43375
81	0.45696
82	0.48042
83	0.50418
84	0.52821
85	0.55253
86	0.57720
87	0.60223
88	0.62763
89	0.65344
90	0.67971
91	0.70645
92	0.73371
93	0.76153
94	0.78997
95	0.81900
96	0.84881
97	0.87935
98	0.91078
99	0.94306
100	0.97639
101	1.01088
102	1.04662
103	1.08367
104	1.12238
105	1.16277
106	1.20523
107	1.24988
108	1.29727
109	1.34777
110	1.40184
111	1.46049
112	1.52465
113	1.59576
114	1.67593
115	1.76871
116	1.87974
117	2.02022
118	2.21696
119	2.56913

What happens when some of the contrasts are positive and some are negative? In Table 3-b, mean contrasts were generated for the following situation:

$$n = 32,\ k = 31,\ r = 8,\ d_6 = +2.00\sigma,\ d_2 = -2.00\sigma$$

Once again it is apparent that with real effects of this size and number, the R-spillover is trivial. This time, however, some spillover occurs at both ends of the scale.

EFFECT OF MARGINAL SIZE EFFECTS. If plots are to be used for screening studies, they must be helpful in detecting real effects that are marginal in size. Let us look at what happens to the estimated contrasts and the R-spillover when the sizes of the effects decrease. To do this, we will hold the other parameters fixed for the moment and use all positive real effects. The contrasts in Table 5 are average values over 5000 runs for this series of situations:

$n = 32$, $k = 31$, $r = 8$, and one table each for $d = +0.50\sigma$ $+1.00\sigma$; $+1.05\sigma$; $+1.15\sigma$; $+1.33\sigma$; $+1.50\sigma$; and $+1.67\sigma$.

As the sizes of the real effects decrease, the probability increases that some real contrasts will be located at ranks other than the first eight positive ones where they were purposely placed for this simulation. Among the intended error contrasts, the more R-spillover at any rank, the further the expected contrast (mean of 5000 runs) at that rank deviates from the e.v.n.o.s. for the number of error contrasts, i.e., 23.

TABLE 5. CHANGES IN RESULTS AS A FUNCTION OF THE MAGNITUDE OF REAL EFFECTS

(a)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 8+0-, d=0.50

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.683	44	-1.931	0.179
2	-0.527	87	-1.491	0.135
3	-0.433	103	-1.225	0.118
4	-0.362	136	-1.024	0.108
5	-0.305	180	-0.862	0.101
6	-0.255	214	-0.722	0.098
7	-0.209	270	-0.593	0.093
8	-0.167	301	-0.472	0.091
9	-0.127	345	-0.361	0.088
10	-0.091	370	-0.257	0.087
11	-0.056	443	-0.158	0.086
12	-0.022	513	-0.061	0.085
13	0.012	580	0.035	0.084
14	0.046	643	0.130	0.084
15	0.080	730	0.225	0.084
16	0.113	802	0.319	0.084
17	0.147	959	0.415	0.084
18	0.182	1106	0.514	0.085
19	0.218	1101	0.616	0.085
20	0.254	1287	0.719	0.086
21	0.292	1517	0.825	0.087
22	0.331	1680	0.936	0.089
23	0.372	1817	1.053	0.091
24	0.416	2029	1.175	0.094
25	0.463	2276	1.309	0.098
26	0.516	2481	1.458	0.102
27	0.576	2850	1.630	0.110
28	0.645	3213	1.824	0.118
29	0.727	3573	2.057	0.132
30	0.839	3981	2.374	0.154
31	1.023	4369	2.894	0.206

(b)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 8+0-, d=1.00

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.687	0	-1.944	0.184
2	-0.526	1	-1.488	0.139
3	-0.432	3	-1.222	0.122
4	-0.361	5	-1.020	0.112
5	-0.302	5	-0.854	0.105
6	-0.250	7	-0.708	0.101
7	-0.203	8	-0.573	0.097
8	-0.159	8	-0.450	0.095
9	-0.118	10	-0.332	0.094
10	-0.079	19	-0.222	0.093
11	-0.040	18	-0.113	0.092
12	-0.001	32	-0.004	0.092
13	0.036	50	0.103	0.091
14	0.074	81	0.210	0.091
15	0.114	81	0.321	0.092
16	0.153	116	0.434	0.093
17	0.196	127	0.554	0.094
18	0.241	198	0.680	0.097
19	0.289	276	0.816	0.099
20	0.343	423	0.969	0.102
21	0.403	654	1.139	0.107
22	0.474	1034	1.340	0.112
23	0.556	1662	1.573	0.119
24	0.654	2853	1.851	0.128
25	0.758	3686	2.144	0.139
26	0.859	4335	2.430	0.141
27	0.959	4630	2.713	0.144
28	1.060	4800	2.997	0.148
29	1.171	4918	3.313	0.157
30	1.305	4974	3.690	0.174
31	1.503	4986	4.250	0.215

TABLE 5. (cont'd)

(c)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 8+0-, d=1.05

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.680	1	-1.923	0.180
2	-0.524	1	-1.481	0.136
3	-0.428	1	-1.210	0.117
4	-0.357	3	-1.010	0.108
5	-0.298	1	-0.842	0.102
6	-0.246	1	-0.697	0.099
7	-0.200	4	-0.566	0.096
8	-0.157	7	-0.443	0.094
9	-0.117	11	-0.331	0.094
10	-0.077	11	-0.219	0.092
11	-0.039	17	-0.109	0.092
12	0.000	18	-0.001	0.092
13	0.037	25	0.106	0.092
14	0.075	38	0.212	0.092
15	0.115	50	0.324	0.092
16	0.154	91	0.436	0.092
17	0.197	108	0.556	0.094
18	0.241	134	0.681	0.097
19	0.291	209	0.823	0.100
20	0.346	336	0.979	0.104
21	0.407	532	1.151	0.109
22	0.479	913	1.355	0.115
23	0.571	1658	1.614	0.122
24	0.682	2943	1.930	0.133
25	0.793	3953	2.244	0.140
26	0.899	4443	2.543	0.144
27	1.002	4719	2.834	0.147
28	1.105	4855	3.126	0.149
29	1.216	4942	3.439	0.154
30	1.350	4982	3.819	0.170
31	1.553	4993	4.392	0.215

(d)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 8+0-, d=1.15

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.684	0	-1.934	0.181
2	-0.522	1	-1.477	0.137
3	-0.428	0	-1.209	0.118
4	-0.358	0	-1.011	0.111
5	-0.298	0	-0.843	0.106
6	-0.246	1	-0.697	0.102
7	-0.198	1	-0.561	0.098
8	-0.156	1	-0.440	0.096
9	-0.115	3	-0.325	0.095
10	-0.075	8	-0.212	0.093
11	-0.036	1	-0.102	0.092
12	0.003	9	0.008	0.092
13	0.041	15	0.115	0.092
14	0.079	20	0.223	0.092
15	0.118	29	0.334	0.092
16	0.159	41	0.449	0.094
17	0.202	65	0.570	0.095
18	0.248	85	0.701	0.098
19	0.298	131	0.842	0.101
20	0.354	225	1.000	0.105
21	0.417	363	1.179	0.111
22	0.496	693	1.401	0.119
23	0.599	1395	1.694	0.131
24	0.739	3258	2.091	0.145
25	0.873	4210	2.469	0.150
26	0.990	4688	2.800	0.149
27	1.098	4861	3.105	0.148
28	1.206	4934	3.410	0.149
29	1.316	4971	3.723	0.155
30	1.448	4995	4.095	0.171
31	1.646	4996	4.655	0.215

TABLE 5. (cont'd)

(e)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 8+0-, d=1.25

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.685	0	-1.938	0.183
2	-0.526	0	-1.488	0.138
3	-0.429	0	-1.214	0.120
4	-0.358	0	-1.012	0.110
5	-0.299	0	-0.847	0.104
6	-0.248	0	-0.702	0.099
7	-0.201	0	-0.568	0.096
8	-0.158	0	-0.446	0.095
9	-0.117	0	-0.331	0.093
10	-0.078	2	-0.221	0.092
11	-0.040	1	-0.113	0.092
12	-0.001	7	-0.003	0.092
13	0.037	10	0.106	0.092
14	0.076	14	0.214	0.093
15	0.114	14	0.323	0.094
16	0.156	10	0.442	0.095
17	0.200	26	0.566	0.097
18	0.246	41	0.697	0.100
19	0.296	55	0.837	0.104
20	0.353	126	0.998	0.108
21	0.421	215	1.191	0.116
22	0.507	464	1.433	0.125
23	0.625	1141	1.767	0.142
24	0.815	3629	2.304	0.162
25	0.963	4537	2.724	0.158
26	1.085	4815	3.069	0.152
27	1.196	4929	3.382	0.153
28	1.304	4974	3.687	0.151
29	1.417	4995	4.007	0.157
30	1.550	4996	4.384	0.171
31	1.756	4999	4.967	0.218

(f)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 8+0-, d=1.33

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.682	0	-1.930	0.181
2	-0.525	0	-1.486	0.136
3	-0.432	0	-1.222	0.120
4	-0.361	0	-1.022	0.111
5	-0.303	0	-0.857	0.105
6	-0.252	0	-0.713	0.101
7	-0.203	2	-0.576	0.098
8	-0.159	0	-0.450	0.096
9	-0.117	1	-0.332	0.093
10	-0.078	1	-0.222	0.092
11	-0.038	3	-0.108	0.091
12	0.001	1	0.003	0.091
13	0.039	3	0.111	0.092
14	0.078	2	0.220	0.093
15	0.117	9	0.331	0.094
16	0.158	6	0.447	0.094
17	0.201	7	0.568	0.097
18	0.247	12	0.700	0.099
19	0.299	54	0.845	0.102
20	0.359	62	1.015	0.108
21	0.426	140	1.206	0.115
22	0.514	285	1.454	0.128
23	0.641	950	1.812	0.149
24	0.876	3939	2.478	0.173
25	1.042	4694	2.946	0.165
26	1.172	4889	3.316	0.156
27	1.282	4960	3.626	0.153
28	1.391	4984	3.935	0.153
29	1.501	4999	4.244	0.160
30	1.633	4998	4.619	0.174
31	1.837	4999	5.195	0.219

TABLE 5. (cont'd)

(g)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 8+0-, d=1.50

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.681	0	-1.926	0.185
2	-0.522	0	-1.478	0.136
3	-0.429	0	-1.214	0.119
4	-0.358	0	-1.013	0.110
5	-0.298	0	-0.843	0.104
6	-0.247	0	-0.699	0.099
7	-0.201	0	-0.568	0.097
8	-0.157	0	-0.444	0.095
9	-0.117	1	-0.330	0.093
10	-0.077	0	-0.217	0.093
11	-0.038	0	-0.108	0.092
12	0.000	0	0.000	0.092
13	0.038	0	0.108	0.092
14	0.077	0	0.217	0.092
15	0.116	0	0.328	0.093
16	0.158	1	0.447	0.094
17	0.202	5	0.571	0.095
18	0.249	6	0.704	0.099
19	0.300	7	0.848	0.103
20	0.359	16	1.017	0.109
21	0.429	47	1.214	0.118
22	0.520	97	1.470	0.134
23	0.664	460	1.879	0.163
24	1.021	4498	2.887	0.192
25	1.205	4896	3.408	0.168
26	1.335	4975	3.775	0.157
27	1.448	4993	4.096	0.152
28	1.554	4998	4.396	0.152
29	1.666	5000	4.712	0.159
30	1.802	5000	5.098	0.177
31	2.004	5000	5.669	0.219

(h)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 8+0-, d=1.67

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.683	0	-1.931	0.182
2	-0.527	0	-1.490	0.137
3	-0.429	0	-1.215	0.119
4	-0.358	0	-1.014	0.111
5	-0.300	0	-0.849	0.105
6	-0.249	0	-0.704	0.101
7	-0.202	0	-0.572	0.097
8	-0.159	0	-0.450	0.094
9	-0.118	0	-0.334	0.092
10	-0.079	0	-0.224	0.091
11	-0.039	0	-0.111	0.091
12	-0.001	0	-0.002	0.091
13	0.038	0	0.107	0.091
14	0.076	0	0.216	0.092
15	0.116	0	0.327	0.093
16	0.157	1	0.443	0.095
17	0.201	0	0.567	0.097
18	0.247	2	0.697	0.100
19	0.298	2	0.843	0.105
20	0.357	2	1.010	0.111
21	0.428	8	1.211	0.120
22	0.522	40	1.476	0.138
23	0.675	217	1.910	0.177
24	1.174	4754	3.322	0.206
25	1.368	4981	3.868	0.171
26	1.502	4994	4.249	0.155
27	1.616	4999	4.571	0.152
28	1.725	5000	4.878	0.152
29	1.837	5000	5.197	0.159
30	1.976	5000	5.588	0.176
31	2.176	5000	6.156	0.218

This can be seen in an examination of the 23 error contrasts in Table 5-b (in which the eight real effects are all of size, $d_e = 1\sigma$). They are no longer symmetrical about rank 12. If we compare them to the 23 e.v.n.o.s. in Table 4, we see that the contrasts closest in rank to the real effects, rank 23, is approximately 0.80 of the expected value and at rank 22, 0.90.

With the smaller real effects, the degrading of the expected contrasts from the correct number of e.v.n.o.s. is more apparent. At ranks 23, 22, and 21 in Table 5-b, the spillover of real effects into those intended-to-be error ranks was appr ximately 33%, 21%, and 14% respectively. That last value means that in one ot seven experiments with 31 contrasts and eight real effects of size d_e = +1.00s, the eleventh largest effect will be real. Conversely, 7% of the time, the fifth largest ranking contrast will not be real.

A summary table, Table 6, provides an overview of the data in Tables 5-a through 5-h.

TABLE 6. SPILLOVER OF REAL AND ERROR CONTRASTS AS A FUNCTION OF EFFECT SIZE
(Situation n = 32, k = 31, r = 8, all positive)

Effect Size ($d\ s_c$)	Effect Size ($d\ \sigma$)	Rank Where R-Spillover Exceeds this Percent 5%	10%	15%	20%	25%	Percent E-Spillover at Rank #24	#25	#26	#27
4.72	1.67	x	x	x	x	x	4	x	x	x
4.24	1.50	23	x	x	x	x	11	x	x	x
3.76	1.33	22	23	x	x	x	21	6	2	1
3.53	1.25	22	23	23	23	x	28	10	3	1
3.25	1.15	21	22	23	23	23	34	15	6	3
2.97	1.05	20	21	22	23	23	40	22	11	5
2.82	1.00	19	21	22	22	23	44	26	13	7
1.41	0.50	7	12	15	18	20	59	54	50	43

Table 6 shows, for example, that among 31 contrasts, of which eight are real, if all real effects are of size $d = 1.15\sigma$ (or $3.25s_c$), we might expect to find one of them, not among the eight largest contrasts of the ordered values, but ranking tenth in size more than 10% of the time. Conversely, 15% of the time, the seventh largest contrast would actually be an error contrast. Since the numbers in Table 6 are all dependent on the size of the experiment and the number and sizes of the real effects, their value to the reader is only to show that caution is required when viewing a set of ordered contrasts and assuming the largest are the real effects, even when they deviate from the null line.

EFFECT OF HAVING MIXED POSITIVE AND NEGATIVE EFFECTS. Let us increase the realism of our simulated data by introducing some negative real effects as well. We have seen what happens when all real effects are bunched at the positive end. The data in Tables 7-a through 7-d allows us to see how the contrasts and the spillover of real and error contrasts into unintended locations change as the proportion of negative effects change. For this table, the following situations were generated.

$n = 32$, $k = 31$, $r = 8$, $d = 1.00\sigma$, with 1, 2, 3, or 4 of the eight real effects being negative.

Scanning the four tables that make up Table 7, one can see that more E-spillover and R-spillover occur at that end of the ordered contrasts where more real effects are intended to be located. For example, in Table 7-a, at the negative end where only one real negative effect was added, 11% of the runs would have real contrasts located at the first adjacent error rank, #2. At the other end, where seven positive real contrasts were added, 32% of the runs would have real contrasts located at the first adjacent error rank, #24.

Furthermore, the 23 mean contrasts in the ranks intended to contain error contrasts are no longer symmetrical as the expected values in Table 4 are. Instead, they are asymmetrical when the number of positive and negative real effects are uneven and are more degraded from their expected values on the end where the most real effects were originally added.

TABLE 7. RESULTS WITH MIXED POSITIVE AND NEGATIVE EFFECTS

(a)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 7+1-, d=1.00

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-1.044	3905	-2.953	0.296
2	-0.657	552	-1.858	0.165
3	-0.517	225	-1.462	0.132
4	-0.426	130	-1.205	0.119
5	-0.356	64	-1.007	0.111
6	-0.299	34	-0.847	0.105
7	-0.249	39	-0.705	0.100
8	-0.202	21	-0.571	0.097
9	-0.158	31	-0.447	0.094
10	-0.117	12	-0.330	0.093
11	-0.077	20	-0.219	0.092
12	-0.039	29	-0.109	0.091
13	0.000	35	-0.001	0.090
14	0.038	42	0.107	0.091
15	0.077	50	0.217	0.091
16	0.115	74	0.326	0.091
17	0.156	103	0.441	0.093
18	0.198	127	0.559	0.094
19	0.243	182	0.688	0.097
20	0.292	265	0.827	0.100
21	0.345	408	0.976	0.104
22	0.404	603	1.143	0.107
23	0.474	892	1.340	0.114
24	0.561	1610	1.588	0.124
25	0.670	2873	1.895	0.136
26	0.783	3777	2.215	0.145
27	0.896	4411	2.534	0.150
28	1.008	4708	2.852	0.153
29	1.125	4858	3.181	0.162
30	1.266	4942	3.581	0.173
31	1.479	4978	4.182	0.216

(b)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 6+2-, d=1.00

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-1.203	4682	-3.403	0.277
2	-0.874	3445	-2.471	0.218
3	-0.627	869	-1.775	0.154
4	-0.506	391	-1.432	0.130
5	-0.421	215	-1.191	0.116
6	-0.353	126	-0.999	0.108
7	-0.295	98	-0.835	0.103
8	-0.244	59	-0.691	0.099
9	-0.199	34	-0.562	0.096
10	-0.155	31	-0.439	0.094
11	-0.115	30	-0.326	0.093
12	-0.076	29	-0.216	0.092
13	-0.038	25	-0.107	0.092
14	0.001	37	0.002	0.092
15	0.039	35	0.110	0.092
16	0.078	50	0.220	0.092
17	0.117	67	0.330	0.095
18	0.157	79	0.443	0.096
19	0.200	113	0.565	0.098
20	0.245	193	0.693	0.100
21	0.294	247	0.831	0.102
22	0.348	363	0.984	0.106
23	0.411	545	1.163	0.111
24	0.484	840	1.369	0.117
25	0.575	1475	1.627	0.128
26	0.693	2885	1.960	0.143
27	0.821	3916	2.323	0.155
28	0.948	4469	2.683	0.159
29	1.081	4757	3.057	0.168
30	1.232	4916	3.483	0.184
31	1.450	4979	4.102	0.227

TABLE 7 (cont'd)

(c)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 5+3-, d=1.00

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-1.300	4864	-3.676	0.258
2	-1.017	4404	-2.876	0.217
3	-0.797	3191	-2.254	0.183
4	-0.614	1130	-1.735	0.143
5	-0.499	530	-1.411	0.125
6	-0.418	301	-1.182	0.114
7	-0.352	177	-0.997	0.106
8	-0.296	125	-0.838	0.102
9	-0.246	67	-0.695	0.099
10	-0.200	64	-0.565	0.096
11	-0.156	46	-0.442	0.094
12	-0.116	47	-0.327	0.093
13	-0.076	20	-0.215	0.092
14	-0.038	31	-0.107	0.091
15	0.000	27	0.001	0.091
16	0.038	40	0.108	0.091
17	0.076	37	0.216	0.092
18	0.115	63	0.326	0.092
19	0.156	77	0.442	0.094
20	0.199	98	0.562	0.096
21	0.245	132	0.692	0.098
22	0.294	213	0.833	0.101
23	0.348	286	0.984	0.105
24	0.413	485	1.168	0.112
25	0.490	765	1.386	0.121
26	0.586	1390	1.657	0.133
27	0.719	2986	2.033	0.153
28	0.863	3975	2.442	0.166
29	1.018	4608	2.878	0.179
30	1.183	4861	3.345	0.194
31	1.414	4960	4.000	0.238

(d)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 4+4-, d=1.00

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-1.370	4931	-3.876	0.244
2	-1.118	4746	-3.163	0.203
3	-0.929	4232	-2.627	0.184
4	-0.755	3020	-2.136	0.165
5	-0.602	1273	-1.702	0.139
6	-0.496	685	-1.403	0.121
7	-0.416	370	-1.178	0.110
8	-0.350	235	-0.991	0.104
9	-0.295	145	-0.834	0.099
10	-0.244	112	-0.691	0.095
11	-0.199	93	-0.563	0.094
12	-0.157	54	-0.443	0.093
13	-0.116	44	-0.329	0.092
14	-0.077	35	-0.218	0.092
15	-0.039	31	-0.110	0.091
16	-0.001	27	-0.002	0.091
17	0.038	30	0.106	0.092
18	0.075	33	0.214	0.092
19	0.116	55	0.328	0.093
20	0.156	54	0.441	0.094
21	0.199	73	0.562	0.097
22	0.245	104	0.694	0.099
23	0.296	160	0.836	0.101
24	0.350	248	0.990	0.105
25	0.416	410	1.175	0.113
26	0.495	694	1.401	0.123
27	0.596	1196	1.687	0.137
28	0.754	3090	2.132	0.166
29	0.924	4193	2.613	0.187
30	1.110	4704	3.139	0.205
31	1.365	4923	3.861	0.240

In a real experiment the data will neither be as smooth as this aggregate data nor will we know how many effects are real nor their sign. However, we can expect fewer spillovers on the end where the fewer obvious effects are visible.

EFFECT OF MIXING THE SIZES OF THE REAL EFFECT. Up to this point, in order to simplify the discussion, we have used simulations in which all real effects were of the same size. In practice this is unlikely to occur. Therefore, the information in these tables are illustrative at best and of limited practical value.

Let us see how the contrasts and spillover patterns change when the real effects are of different magnitudes. For this, let us examine this situation:

$n = 32$, $k = 31$, $r = 8$, d (one for each r) = $+1.00\sigma$, $+1.00\sigma$, $+1.33\sigma$, $+1.33\sigma$, $+1.67\sigma$, $+1.67\sigma$, $+2.00\sigma$, $+2.00\sigma$.

The results are shown in Table 8.

It would be a valuable tool if a limited number of stylized tables of the type shown in this report could be prepared from which an investigator might draw data which he could then combine to fit his particular situation. Could we have predicted these results from the tables that we've already generated? To what extent does the data in Table 8 (where the average of the eight effects sizes equals 1.5) resemble the data in Table 5-g where $d_e = +1.5\sigma$? To what extent does the data in Table 8 resemble the averages of values from four sets of tables (Table 5-b, 5-f, 5-h, and 3-a) where all eight of the real effects are of size 1.00σ, 1.33σ, 1.67σ, and 2.00σ, respectively?

Fewer real effects spilled into the lower ranks when all eight effects were of size 1.5σ (Table 5-g) than when they were mixed, but with sizes averaging 1.5σ (Table 8). The smaller size real effects exert a much stronger effect on the data than the larger ones in the mixed data.

TABLE 8. RESULTS WHERE THE SIZES OF THE REAL EFFECTS ARE MIXED

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r=MIXED, d=1.00

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.681	0	-1.927	0.181
2	-0.525	1	-1.484	0.140
3	-0.430	0	-1.216	0.122
4	-0.359	1	-1.016	0.111
5	-0.300	1	-0.849	0.106
6	-0.248	2	-0.702	0.102
7	-0.201	0	-0.569	0.098
8	-0.157	2	-0.444	0.096
9	-0.116	2	-0.327	0.095
10	-0.076	6	-0.216	0.093
11	-0.037	4	-0.105	0.093
12	0.000	10	0.000	0.091
13	0.039	16	0.109	0.092
14	0.076	16	0.216	0.092
15	0.116	29	0.328	0.093
16	0.158	21	0.446	0.096
17	0.200	42	0.566	0.098
18	0.246	46	0.696	0.100
19	0.298	91	0.842	0.104
20	0.354	153	1.000	0.108
21	0.421	246	1.190	0.115
22	0.505	433	1.427	0.127
23	0.625	1042	1.767	0.149
24	0.827	3398	2.339	0.186
25	1.037	4554	2.934	0.194
26	1.235	4900	3.492	0.186
27	1.411	4985	3.992	0.182
28	1.589	4999	4.495	0.183
29	1.775	5000	5.020	0.191
30	1.979	5000	5.598	0.206
31	2.256	5000	6.380	0.254

Nor does the average of results from four sets of data (Tables 5-f, 5-h, and 3-a) that bracket the mixed data in Table 8 adequately represent the results from the mixed data. Efforts to improve this resemblance using log and z-score transformations were also inadequate. Here again the smaller effects have a stronger influence on the degree of spillover and the degradation in contrast estimates. In fact, the mixed data fell between data where all effects were of size 1σ and 1.33σ, illustrating again the greater impact of the marginal effects.

Although not totally unexpected, these results indicate that no simple relationship exists. The idea of having a limited set of tables from which an investigator can select and recombine information to better understand his own particular results is not likely to become possible without considerable more work, if at all.

EFFECT OF A DIFFERENT NUMBER OF REAL EFFECTS. Zahn (1975b) found that as the number of real effects increased, there was a drop in the detection rate. Let us examine the effect of changing r while holding the other parameters constant, for the following situations:

$n = 32$, $k = 31$, $d_r = +1.25\sigma$, $r = 4; 5; 6; 7; 9; 10; 12;$ or 16

The mean results from 5000 runs are given in Table 9.

As the number of real effects increase, the greater the chances are that one or more of the larger contrasts are not real.

EFFECT OF INCREASING THE SIZE OF THE EXPERIMENT. Since increasing n in an experiment decreases the size of the error variance, the sensitivity of the test for significance increases and more real effects are detectable. The same principle should apply to this plot data.

The following situations are shown in Table 10.

$r = 8$, $d_s = +1.00\sigma$, $n = 32, 64,$ or 128, and $k = (n - 1)$.

TABLE 9. CHANGES IN RESULTS AS A FUNCTION OF THE NUMBER OF REAL EFFECTS

(a)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 4+0-, d=1.25

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.711	0	-2.011	0.179
2	-0.552	0	-1.561	0.136
3	-0.461	0	-1.305	0.117
4	-0.394	0	-1.114	0.108
5	-0.338	1	-0.956	0.101
6	-0.289	0	-0.817	0.096
7	-0.245	0	-0.693	0.093
8	-0.205	0	-0.581	0.090
9	-0.169	0	-0.477	0.088
10	-0.134	0	-0.378	0.087
11	-0.099	0	-0.279	0.086
12	-0.065	0	-0.184	0.085
13	-0.033	2	-0.093	0.085
14	0.000	1	0.001	0.084
15	0.033	3	0.094	0.085
16	0.067	4	0.189	0.085
17	0.100	5	0.283	0.085
18	0.135	7	0.381	0.086
19	0.171	14	0.483	0.087
20	0.209	23	0.591	0.089
21	0.249	17	0.705	0.092
22	0.292	31	0.825	0.095
23	0.339	42	0.958	0.099
24	0.394	66	1.114	0.104
25	0.461	126	1.304	0.113
26	0.545	284	1.542	0.125
27	0.671	765	1.899	0.150
28	0.938	3894	2.653	0.198
29	1.158	4779	3.274	0.202
30	1.359	4945	3.845	0.210
31	1.614	4991	4.564	0.247

(b)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 5+0-, d=1.25

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.699	0	-1.977	0.179
2	-0.545	0	-1.543	0.133
3	-0.454	0	-1.285	0.117
4	-0.383	0	-1.083	0.106
5	-0.327	0	-0.926	0.101
6	-0.278	0	-0.786	0.095
7	-0.233	1	-0.659	0.091
8	-0.193	0	-0.545	0.089
9	-0.154	0	-0.435	0.088
10	-0.118	0	-0.334	0.087
11	-0.083	1	-0.235	0.086
12	-0.049	2	-0.138	0.085
13	-0.015	4	-0.042	0.085
14	0.021	2	0.058	0.084
15	0.053	6	0.151	0.085
16	0.088	6	0.248	0.085
17	0.122	4	0.345	0.086
18	0.159	11	0.449	0.088
19	0.198	12	0.559	0.090
20	0.238	17	0.674	0.092
21	0.282	38	0.797	0.096
22	0.330	55	0.933	0.099
23	0.384	78	1.087	0.104
24	0.450	140	1.274	0.114
25	0.535	320	1.513	0.125
26	0.659	915	1.864	0.147
27	0.892	3836	2.524	0.187
28	1.080	4661	3.053	0.186
29	1.245	4913	3.521	0.184
30	1.421	4979	4.018	0.192
31	1.654	4999	4.678	0.233

TABLE 9. (cont'd)

(c)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 6+0-, d=1.25

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.694	0	-1.964	0.179
2	-0.537	0	-1.520	0.134
3	-0.445	0	-1.260	0.117
4	-0.376	1	-1.062	0.108
5	-0.317	0	-0.898	0.101
6	-0.267	1	-0.755	0.096
7	-0.221	1	-0.626	0.094
8	-0.181	0	-0.511	0.092
9	-0.142	1	-0.402	0.091
10	-0.104	0	-0.295	0.089
11	-0.069	5	-0.194	0.088
12	-0.034	1	-0.095	0.087
13	0.002	3	0.005	0.087
14	0.036	8	0.102	0.087
15	0.071	3	0.202	0.088
16	0.107	3	0.303	0.088
17	0.144	16	0.408	0.090
18	0.183	14	0.518	0.091
19	0.224	27	0.633	0.094
20	0.269	26	0.762	0.096
21	0.318	49	0.901	0.100
22	0.375	73	1.062	0.104
23	0.443	183	1.252	0.112
24	0.528	386	1.493	0.124
25	0.647	972	1.830	0.142
26	0.859	3769	2.428	0.173
27	1.030	4621	2.912	0.173
28	1.180	4881	3.337	0.172
29	1.321	4967	3.735	0.173
30	1.475	4992	4.171	0.184
31	1.698	4997	4.802	0.229

(d)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 7+0-, d=1.25

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.685	0	-1.936	0.180
2	-0.531	0	-1.503	0.133
3	-0.437	0	-1.236	0.116
4	-0.368	0	-1.040	0.107
5	-0.310	0	-0.876	0.101
6	-0.258	0	-0.730	0.098
7	-0.212	0	-0.599	0.095
8	-0.170	0	-0.480	0.093
9	-0.129	0	-0.365	0.092
10	-0.091	3	-0.257	0.091
11	-0.053	2	-0.149	0.091
12	-0.015	4	-0.043	0.090
13	0.021	3	0.060	0.091
14	0.058	4	0.165	0.091
15	0.096	6	0.272	0.091
16	0.135	12	0.383	0.092
17	0.175	8	0.495	0.094
18	0.218	21	0.617	0.095
19	0.263	39	0.744	0.097
20	0.312	81	0.883	0.100
21	0.369	114	1.043	0.105
22	0.435	226	1.230	0.113
23	0.517	396	1.463	0.124
24	0.635	1113	1.796	0.143
25	0.833	3648	2.356	0.166
26	0.993	4556	2.808	0.165
27	1.127	4854	3.188	0.162
28	1.250	4935	3.535	0.163
29	1.373	4984	3.883	0.168
30	1.516	4993	4.289	0.181
31	1.723	4998	4.875	0.222

TABLE 9. (cont'd)

(e)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 9+0-, d=1.25

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.675	0	-1.908	0.183
2	-0.516	0	-1.459	0.138
3	-0.419	0	-1.186	0.121
4	-0.348	0	-0.983	0.111
5	-0.287	0	-0.813	0.104
6	-0.235	0	-0.664	0.100
7	-0.187	0	-0.529	0.098
8	-0.143	0	-0.404	0.097
9	-0.101	1	-0.287	0.095
10	-0.060	2	-0.171	0.093
11	-0.020	6	-0.058	0.093
12	0.020	8	0.057	0.093
13	0.060	6	0.169	0.092
14	0.100	13	0.283	0.093
15	0.143	16	0.403	0.095
16	0.187	33	0.528	0.097
17	0.235	37	0.664	0.100
18	0.286	66	0.810	0.103
19	0.344	113	0.974	0.109
20	0.415	246	1.172	0.117
21	0.500	494	1.416	0.128
22	0.620	1175	1.753	0.143
23	0.798	3643	2.258	0.155
24	0.941	4506	2.661	0.150
25	1.052	4796	2.976	0.145
26	1.154	4910	3.265	0.142
27	1.252	4959	3.541	0.141
28	1.350	4978	3.818	0.145
29	1.456	4994	4.119	0.151
30	1.584	4998	4.480	0.169
31	1.781	5000	5.038	0.214

(f)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r=10+0-, d=1.25

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.669	0	-1.891	0.183
2	-0.509	0	-1.439	0.139
3	-0.409	0	-1.157	0.122
4	-0.338	0	-0.956	0.113
5	-0.278	0	-0.785	0.109
6	-0.223	0	-0.632	0.104
7	-0.174	1	-0.493	0.101
8	-0.128	1	-0.363	0.099
9	-0.084	1	-0.238	0.097
10	-0.041	5	-0.115	0.096
11	0.001	6	0.002	0.096
12	0.043	6	0.121	0.096
13	0.085	6	0.240	0.097
14	0.129	14	0.365	0.099
15	0.174	34	0.492	0.100
16	0.222	57	0.627	0.102
17	0.275	69	0.777	0.106
18	0.335	111	0.947	0.110
19	0.404	244	1.142	0.118
20	0.491	510	1.389	0.128
21	0.608	1278	1.720	0.141
22	0.778	3560	2.202	0.152
23	0.914	4465	2.585	0.147
24	1.026	4795	2.901	0.139
25	1.118	4926	3.162	0.137
26	1.205	4953	3.407	0.136
27	1.291	4975	3.652	0.136
28	1.381	4992	3.905	0.139
29	1.481	4995	4.188	0.147
30	1.603	4996	4.534	0.163
31	1.794	5000	5.074	0.206

TABLE 9. (cont'd)

(g)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r=12+0-, d=1.25

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.653	0	-1.846	0.188
2	-0.489	0	-1.383	0.142
3	-0.388	1	-1.099	0.125
4	-0.312	0	-0.884	0.117
5	-0.250	1	-0.708	0.110
6	-0.193	2	-0.546	0.105
7	-0.142	0	-0.401	0.103
8	-0.093	0	-0.264	0.101
9	-0.047	7	-0.132	0.100
10	0.000	7	0.000	0.099
11	0.046	13	0.131	0.100
12	0.093	14	0.264	0.100
13	0.143	31	0.403	0.101
14	0.193	49	0.545	0.104
15	0.248	86	0.701	0.107
16	0.309	119	0.875	0.111
17	0.380	233	1.076	0.117
18	0.467	518	1.322	0.127
19	0.582	1352	1.645	0.140
20	0.748	3525	2.115	0.149
21	0.877	4487	2.479	0.145
22	0.979	4768	2.768	0.137
23	1.066	4888	3.016	0.129
24	1.144	4953	3.235	0.127
25	1.217	4964	3.442	0.125
26	1.287	4986	3.640	0.126
27	1.360	4997	3.848	0.129
28	1.441	5000	4.075	0.132
29	1.531	4999	4.331	0.142
30	1.649	5000	4.663	0.158
31	1.827	5000	5.168	0.201

(h)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r=16+0-, d=1.25

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.613	0	-1.733	0.191
2	-0.442	0	-1.249	0.150
3	-0.334	0	-0.946	0.132
4	-0.252	0	-0.711	0.124
5	-0.183	2	-0.517	0.118
6	-0.118	7	-0.333	0.114
7	-0.058	10	-0.165	0.111
8	-0.002	18	-0.005	0.109
9	0.057	26	0.161	0.110
10	0.117	37	0.330	0.112
11	0.181	81	0.511	0.115
12	0.252	157	0.711	0.120
13	0.329	279	0.930	0.126
14	0.422	584	1.195	0.134
15	0.540	1409	1.527	0.141
16	0.698	3549	1.974	0.140
17	0.816	4394	2.308	0.131
18	0.909	4735	2.571	0.123
19	0.986	4862	2.788	0.119
20	1.052	4932	2.976	0.114
21	1.114	4962	3.151	0.111
22	1.169	4983	3.308	0.110
23	1.225	4988	3.464	0.109
24	1.277	4994	3.613	0.109
25	1.332	4997	3.766	0.109
26	1.389	4996	3.928	0.111
27	1.449	5000	4.098	0.114
28	1.517	4998	4.291	0.120
29	1.597	5000	4.517	0.127
30	1.701	5000	4.810	0.146
31	1.874	5000	5.300	0.194

TABLE 10. CHANGES IN RESULTS AS A FUNCTION OF THE SIZE OF THE EXPERIMENT

(a)

NORMAL PLOT DATA

SITUATION: n= 32, k= 31, r= 8+0-, d=1.00

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.686	0	-1.940	0.182
2	-0.525	0	-1.485	0.138
3	-0.430	0	-1.215	0.121
4	-0.359	1	-1.014	0.112
5	-0.299	5	-0.847	0.106
6	-0.248	6	-0.700	0.101
7	-0.202	4	-0.570	0.098
8	-0.158	9	-0.446	0.095
9	-0.117	10	-0.331	0.094
10	-0.078	14	-0.220	0.093
11	-0.039	30	-0.111	0.092
12	-0.001	28	-0.004	0.091
13	0.036	40	0.102	0.091
14	0.074	62	0.209	0.092
15	0.114	83	0.323	0.092
16	0.154	109	0.436	0.092
17	0.197	156	0.558	0.094
18	0.242	206	0.684	0.096
19	0.290	258	0.820	0.099
20	0.343	444	0.971	0.102
21	0.403	670	1.139	0.106
22	0.472	1050	1.335	0.111
23	0.555	1754	1.568	0.119
24	0.652	2781	1.843	0.131
25	0.755	3694	2.135	0.138
26	0.857	4288	2.423	0.141
27	0.956	4618	2.704	0.143
28	1.058	4810	2.993	0.146
29	1.170	4918	3.308	0.153
30	1.303	4959	3.684	0.171
31	1.506	4993	4.260	0.218

(b)

NORMAL PLOT DATA

SITUATION: n= 64, k= 63, r= 8+0-, d=1.00

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.570	0	-2.281	0.114
2	-0.473	0	-1.893	0.082
3	-0.419	0	-1.675	0.071
4	-0.378	0	-1.512	0.064
5	-0.346	0	-1.383	0.060
6	-0.318	0	-1.274	0.058
7	-0.294	0	-1.176	0.055
8	-0.272	0	-1.089	0.053
9	-0.252	0	-1.009	0.051
10	-0.234	0	-0.935	0.050
11	-0.217	0	-0.867	0.048
12	-0.201	0	-0.804	0.047
13	-0.186	0	-0.743	0.046
14	-0.171	0	-0.684	0.046
15	-0.157	0	-0.627	0.045
16	-0.144	1	-0.574	0.045
17	-0.130	0	-0.522	0.045
18	-0.117	0	-0.470	0.045
19	-0.105	0	-0.421	0.044
20	-0.093	0	-0.371	0.044
21	-0.081	0	-0.324	0.044
22	-0.069	0	-0.278	0.043
23	-0.058	1	-0.230	0.043
24	-0.046	0	-0.184	0.043
25	-0.034	0	-0.137	0.042
26	-0.023	0	-0.090	0.042
27	-0.011	0	-0.044	0.042
28	0.000	0	-0.001	0.042
29	0.011	0	0.044	0.042
30	0.023	0	0.090	0.043
31	0.034	1	0.136	0.043
32	0.045	0	0.182	0.042
33	0.057	3	0.229	0.043
34	0.069	0	0.276	0.043
35	0.081	1	0.322	0.043
36	0.093	0	0.370	0.043
37	0.105	4	0.420	0.043
38	0.118	0	0.471	0.044
39	0.131	4	0.523	0.044
40	0.143	6	0.573	0.045
41	0.157	4	0.627	0.045
42	0.171	2	0.685	0.046
43	0.186	6	0.744	0.047
44	0.201	7	0.806	0.047
45	0.218	8	0.873	0.049
46	0.235	6	0.940	0.050
47	0.253	12	1.013	0.051
48	0.273	21	1.091	0.053
49	0.294	19	1.176	0.055
50	0.317	43	1.267	0.057
51	0.344	62	1.376	0.061
52	0.376	105	1.505	0.064
53	0.415	161	1.659	0.070
54	0.466	354	1.862	0.079
55	0.539	942	2.158	0.093
56	0.682	3732	2.729	0.110
57	0.795	4645	3.182	0.115
58	0.885	4895	3.540	0.109
59	0.966	4967	3.863	0.106
60	1.040	4993	4.160	0.107
61	1.120	4997	4.479	0.112
62	1.214	4998	4.856	0.123
63	1.354	5000	5.415	0.151

TABLE 10. (cont'd)

(c)

NORMAL PLOT DATA

SITUATION: n=128, k=127, r= 8+0-, d=1.00

#	CONTRAST	# R	STANDARDIZED CONTRAST	STD. DEVIATION
1	-0.454	0	-2.569	0.074
2	-0.392	0	-2.215	0.053
3	-0.357	0	-2.019	0.044
4	-0.332	0	-1.878	0.039
5	-0.312	0	-1.768	0.036
6	-0.296	0	-1.675	0.034
7	-0.282	0	-1.597	0.033
8	-0.270	0	-1.525	0.031
9	-0.258	0	-1.461	0.030
10	-0.248	0	-1.402	0.029
11	-0.238	0	-1.348	0.028
12	-0.229	0	-1.298	0.028
13	-0.221	0	-1.251	0.027
14	-0.213	0	-1.206	0.027
15	-0.206	0	-1.163	0.026
16	-0.199	0	-1.124	0.026
17	-0.192	0	-1.085	0.025
18	-0.185	0	-1.048	0.025
19	-0.179	0	-1.013	0.024
20	-0.173	0	-0.978	0.024
21	-0.167	0	-0.945	0.024
22	-0.161	0	-0.913	0.024
23	-0.156	0	-0.882	0.023
24	-0.150	0	-0.851	0.023
25	-0.145	0	-0.821	0.023
26	-0.140	0	-0.792	0.023
27	-0.135	0	-0.764	0.023
28	-0.130	0	-0.735	0.022
29	-0.125	0	-0.708	0.022
30	-0.121	0	-0.682	0.022
31	-0.116	0	-0.655	0.022
32	-0.111	0	-0.630	0.022
33	-0.107	0	-0.605	0.022
34	-0.102	0	-0.579	0.022
35	-0.098	0	-0.555	0.022
36	-0.094	0	-0.531	0.021
37	-0.089	0	-0.506	0.021
38	-0.085	0	-0.482	0.021
39	-0.081	0	-0.458	0.021
40	-0.077	0	-0.435	0.021
41	-0.073	0	-0.412	0.021
42	-0.069	0	-0.389	0.021
43	-0.065	0	-0.366	0.021
44	-0.061	0	-0.344	0.021
45	-0.057	0	-0.321	0.020
46	-0.053	0	-0.299	0.020
47	-0.049	0	-0.277	0.020
48	-0.045	0	-0.255	0.020
49	-0.041	0	-0.233	0.020
50	-0.037	0	-0.211	0.020
51	-0.034	0	-0.190	0.020
52	-0.030	0	-0.168	0.020
53	-0.026	0	-0.147	0.020
54	-0.022	0	-0.126	0.020
55	-0.018	0	-0.104	0.020
56	-0.015	0	-0.083	0.020
57	-0.011	0	-0.062	0.020
58	-0.007	0	-0.040	0.020
59	-0.003	0	-0.020	0.020
60	0.000	0	0.001	0.020
61	0.004	0	0.022	0.020
62	0.008	0	0.044	0.020
63	0.011	0	0.065	0.020
64	0.015	0	0.086	0.020
65	0.019	0	0.106	0.020
66	0.022	0	0.127	0.020
67	0.026	0	0.148	0.020
68	0.030	0	0.170	0.020
69	0.034	0	0.191	0.020
70	0.038	0	0.212	0.020
71	0.041	0	0.233	0.020
72	0.045	0	0.255	0.020
73	0.049	0	0.277	0.020
74	0.053	0	0.298	0.020
75	0.057	0	0.320	0.020
76	0.060	0	0.342	0.020
77	0.064	0	0.365	0.020
78	0.069	0	0.388	0.020
79	0.073	0	0.410	0.021
80	0.077	0	0.434	0.021
81	0.081	0	0.457	0.021
82	0.085	0	0.480	0.021
83	0.089	0	0.504	0.021
84	0.093	0	0.528	0.021
85	0.098	0	0.553	0.021
86	0.102	0	0.578	0.021
87	0.107	0	0.603	0.022
88	0.111	0	0.628	0.022
89	0.116	0	0.654	0.022
90	0.120	0	0.681	0.022
91	0.125	0	0.708	0.022
92	0.130	0	0.735	0.022
93	0.135	0	0.763	0.022
94	0.140	0	0.791	0.023
95	0.145	0	0.821	0.023
96	0.151	0	0.851	0.023
97	0.156	0	0.882	0.023
98	0.162	0	0.914	0.021
99	0.167	0	0.946	0.024
100	0.173	0	0.979	0.024
101	0.179	0	1.013	0.024
102	0.185	0	1.048	0.025
103	0.192	0	1.085	0.025
104	0.199	0	1.124	0.025
105	0.206	0	1.164	0.026
106	0.213	0	1.208	0.027
107	0.221	0	1.252	0.027
108	0.230	0	1.299	0.028
109	0.238	0	1.349	0.029
110	0.248	0	1.403	0.029
111	0.258	0	1.460	0.030
112	0.270	0	1.525	0.031
113	0.282	0	1.596	0.033
114	0.296	0	1.676	0.034
115	0.313	1	1.769	0.037
116	0.333	1	1.881	0.040
117	0.358	5	2.025	0.046
118	0.392	9	2.219	0.054
119	0.454	79	2.569	0.073
120	0.749	4912	4.235	0.105
121	0.849	4993	4.804	0.086
122	0.917	5000	5.188	0.079
123	0.973	5000	5.505	0.077
124	1.028	5000	5.817	0.078
125	1.086	5000	6.144	0.083
126	1.154	5000	6.525	0.088
127	1.254	5000	7.091	0.103

To make the comparison, let us examine the R-spillover in the first intended error rank adjacent to the rank of the eight positive real effects. The results are shown in Table 11.

TABLE 11. STANDARDIZED CONTRAST DEGRADATION AND R-SPILLOVER IN FIRST ERROR RANK AS A FUNCTION OF n WHERE r = 8 and d_8 = +1.00σ

k = n - 1	31	63	127
1st Error Rank	23	55	119
R-Spillover as 1st Error Rank	34%	18%	2%
Number of Intended Error Ranks Into Which Real Effects Might Fall More Than 5% of the Time	5	2	0
Mean Standardized Contrast	1.568	2.158	2.569
Expected Value	1.929	2.285	2.635
Degradation	19%	6%	2%
R-Spillover	34%	18%	2%

SUMMARY OF NORMAL PLOT CHARACTERISTICS AS A FUNCTION OF N, K, R, R+, AND D. It is apparent from the above data that contrast degradation and R-spillover are functions of all five parameters, n, k, r, R+, and d. In that regard, the results of any experiment are unique.

This suggests that preparing a set of generalizable tables would be a horrendous task and probably far too bulky to be used effectively, if at all. The search for the ideal solution, that is, to find a simple mathematical equation which expresses these relationships, is beyond the scope of this paper.

In Figure 4, however, a two-dimensional plot is given relating number of real effects and sizes of real effects (when all are equal) to R-spillover for n = 32 and k = 31. The results are surprisingly clean and simple, with a few exceptions, possibly due to the unreliability of the numbers and/or an inadequate model of the relationship. Work is ongoing to study this relationship further

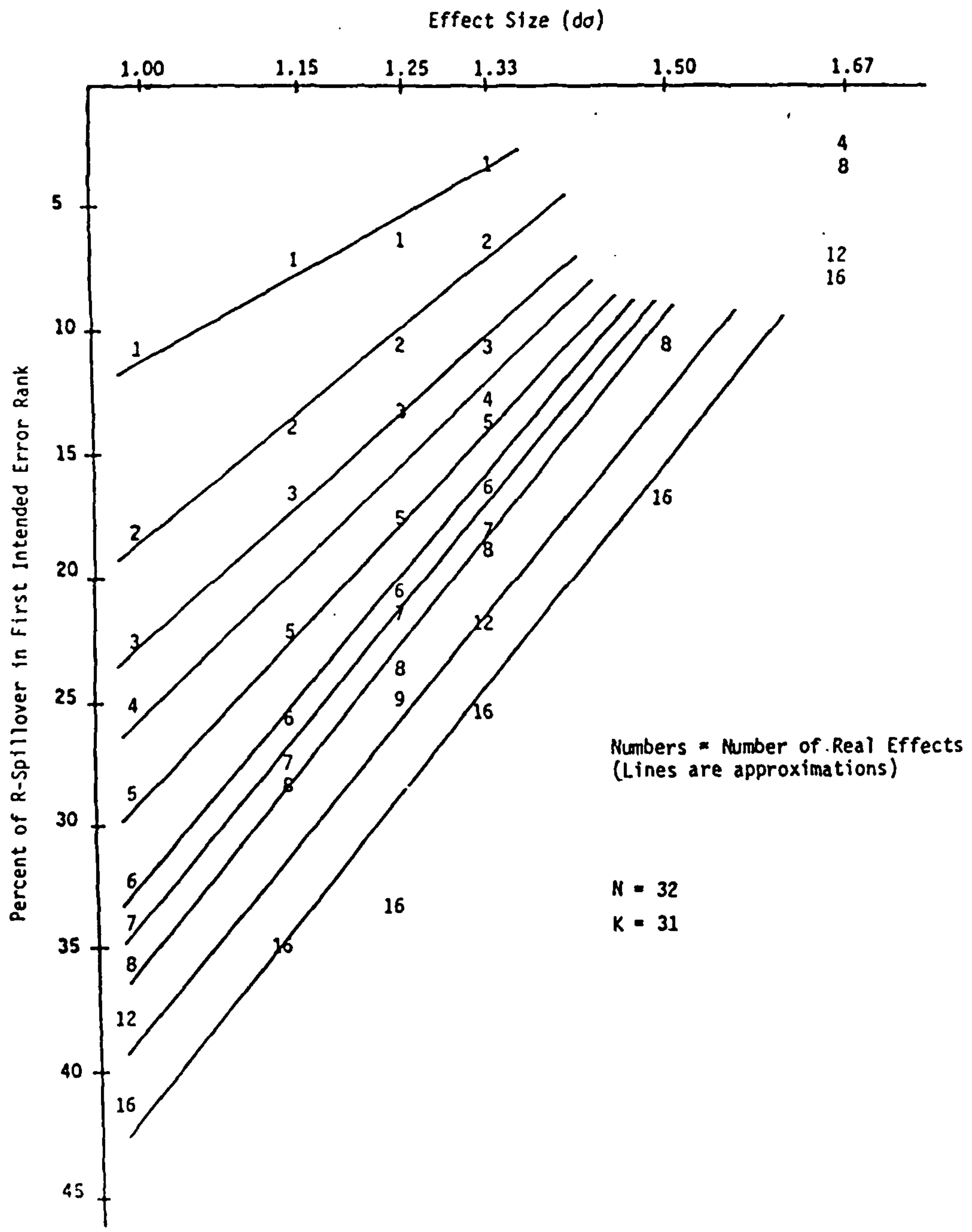


Figure 4. Relating Number of Real Effects, Size of Real Effects, and R-Spillover in First Intended Error Rank.

NORMAL VERSUS HALF-NORMAL PLOT DATA

In a screening experiment, the primary concern is to detect all of the critical factors. Whether these factors have large or small effects is of minor importance in this initial phase, and interactions are only important to the extent that some critical ones may contain factors that do not otherwise show up as a critical main effect. That class of interaction, an "intrinsic" one, is of particular interest. During the screening phase, the investigator will have to weigh the cost-effectiveness of including some false positives in order not to miss some marginal real effects.

Is there a practical difference between normal and half-normal plots as an aid in the detection of real effects? Do they differ in the degree of R-spillover that occurs at ranks that fall at the border of real and error contrasts? There are many who continue to use the half-normal plot as a detection device. Daniel (1976), on the other hand, has more recently expressed a preference for the normal plot, although he tended to use it more as a tool to detect abnormalities in his experimental data. This change occurred because he felt that the signed contrasts of the normal plot provided more information than the absolute contrasts of the half-normal data.

In this section we will compare some advantages and disadvantages of normal and half-normal plotting insofar as the screening process is concerned.

R-SPILLOVER. The data for the following half-normal situations, an aggregate of 5000 runs, are shown in Table 12.

$n = 32$, $k = 31$, $r = 8$, $d_8 = +1.00\sigma$; $+1.25\sigma$; $+1.5\sigma$; or 2.00σ

$n = 64$; $k = 63$; $r = 8$, $d_8 = +1.00\sigma$, $+1.25\sigma$, or 1.50σ

$n = 128$, $k = 127$, $r = 8$, $d_8 = +1.00\sigma$

In Table 13, the R-spillover patterns of these half-normal situations are compared with those of comparable normal situations (data in which the effects were all located at the positive end of the scale).

TABLE 12. HALF-NORMAL PLOT DATA
(Situation: $n = 32$, $k = 31$, $r = 8$, $d = 1.00\sigma$)

(a)

#	BIAS	#EFFECTS	STD DEV	STD BIAS
1	0.0187	30	0.01857	0.0529
2	0.0375	37	0.02560	0.1061
3	0.0567	36	0.03084	0.1604
4	0.0757	43	0.03519	0.2140
5	0.0951	38	0.03916	0.2689
6	0.1149	45	0.04236	0.3250
7	0.1344	41	0.04487	0.3800
8	0.1548	61	0.04770	0.4377
9	0.1764	74	0.05071	0.4989
10	0.1982	66	0.05351	0.5605
11	0.2203	91	0.05575	0.6231
12	0.24[illegible]	94	0.05802	0.6880
13	0.2668	142	0.06021	0.7546
14	0.2916	148	0.06252	0.8247
15	0.3175	170	0.06554	0.8981
16	0.3456	212	0.06773	0.9776
17	0.3751	283	0.07081	1.0610
18	0.4073	348	0.07299	1.1519
19	0.4425	504	0.07649	1.2515
20	0.4815	609	0.08049	1.3620
21	0.5260	840	0.08611	1.4878
22	0.5753	1164	0.09094	1.6272
23	0.6342	1610	0.09968	1.7937
24	0.7060	2443	0.10966	1.9968
25	0.7875	3190	0.12126	2.2274
26	0.8740	3855	0.12926	2.4720
27	0.9661	4380	0.13858	2.7325
28	1.0618	4669	0.14397	3.0031
29	1.1717	4844	0.15569	3.3142
30	1.3036	4947	0.17253	3.6873
31	1.4984	4986	0.20921	4.2381

(b)

#	BIAS	#EFFECTS	STD DEV	STD BIAS
1	0.0184	1	0.01784	0.0522
2	0.0368	3	0.02481	0.1042
3	0.0558	3	0.03029	0.1579
4	0.0752	5	0.03467	0.2128
5	0.0945	2	0.03823	0.2674
6	0.1144	8	0.04107	0.3234
7	0.1338	3	0.04391	0.3784
8	0.1539	4	0.04706	0.4354
9	0.1753	7	0.04972	0.4958
10	0.1970	11	0.05267	0.5571
11	0.2192	12	0.05551	0.6200
12	0.2420	26	0.05756	0.6844
13	0.2665	25	0.06010	0.7539
14	0.2926	37	0.06221	0.8275
15	0.3200	37	0.06530	0.9051
16	0.3500	64	0.06788	0.9900
17	0.3813	72	0.07128	1.0784
18	0.4158	82	0.07502	1.1762
19	0.4541	125	0.07988	1.2843
20	0.4981	231	0.08555	1.4088
21	0.5515	375	0.09194	1.5597
22	0.6171	640	0.10072	1.7453
23	0.7069	1251	0.11554	1.9993
24	0.8390	3178	0.14162	2.3731
25	0.9700	4252	0.15021	2.7435
26	1.0865	4714	0.14978	3.0732
27	1.1993	4897	0.15087	3.3920
28	1.3060	4951	0.15447	3.6940
29	1.4198	4987	0.15757	4.0157
30	1.5516	4997	0.17200	4.3886
31	1.7483	5000	0.20577	4.9450

TABLE 12 (cont'd)
(Situation: n = 64, k = 63, r = 8, d = 1.00σ)

(c)

#	BIAS	#EFFECTS	STD DEV	STD BIAS
1	0.0189	0	0.01797	0.0536
2	0.0377	0	0.02518	0.1067
3	0.0566	0	0.03012	0.1600
4	0.0760	0	0.03445	0.2149
5	0.0956	1	0.03822	0.2703
6	0.1155	0	0.04144	0.3267
7	0.1358	0	0.04431	0.3841
8	0.1563	1	0.04725	0.4420
9	0.1771	2	0.04955	0.5009
10	0.1991	1	0.05185	0.5630
11	0.2218	0	0.05458	0.6272
12	0.2452	0	0.05747	0.6935
13	0.2701	2	0.06044	0.7639
14	0.2956	2	0.06334	0.8360
15	0.3230	3	0.06618	0.9135
16	0.3519	7	0.06906	0.9954
17	0.3839	10	0.07301	1.0859
18	0.4196	13	0.07722	1.1868
19	0.4590	20	0.08193	1.2983
20	0.5055	36	0.08948	1.4297
21	0.5637	88	0.09756	1.5944
22	0.6395	190	0.11271	1.8088
23	0.7574	620	0.13464	2.1421
24	1.0225	4217	0.17780	2.8919
25	1.1959	4842	0.16831	3.3826
26	1.3322	4953	0.15874	3.7682
27	1.4455	4993	0.15461	4.0884
28	1.5544	5000	0.15295	4.3965
29	1.6672	5000	0.15986	4.7155
30	1.7989	4999	0.16929	5.0880
31	1.9989	5000	0.20974	5.6538

(d)

#	BIAS	#EFFECTS	STD DEV	STD BIAS
1	0.0190	0	0.01803	0.0538
2	0.0383	0	0.02511	0.1083
3	0.0571	0	0.03006	0.1616
4	0.0761	0	0.03424	0.2152
5	0.0952	0	0.03812	0.2694
6	0.1156	0	0.04118	0.3270
7	0.1354	0	0.04404	0.3831
8	0.1562	0	0.04749	0.4419
9	0.1770	0	0.04978	0.5007
10	0.1990	0	0.05253	0.5629
11	0.2220	0	0.05504	0.6279
12	0.2455	0	0.05771	0.6945
13	0.2698	0	0.06009	0.7631
14	0.2958	0	0.06246	0.8367
15	0.3240	0	0.06528	0.9164
16	0.3532	0	0.06859	0.9990
17	0.3850	0	0.07253	1.0891
18	0.4210	0	0.07738	1.1909
19	0.4611	0	0.08227	1.3041
20	0.5077	0	0.08884	1.4359
21	0.5660	0	0.09842	1.6009
22	0.6460	3	0.11577	1.8272
23	0.7790	27	0.15223	2.2034
24	1.5027	4972	0.20994	4.2503
25	1.7000	4999	0.17458	4.8084
26	1.8332	5000	0.16083	5.1850
27	1.9478	5000	0.15484	5.5093
28	2.0576	5000	0.15513	5.8198
29	2.1714	5000	0.15927	6.1416
30	2.3084	4999	0.17177	6.5291
31	2.5035	5000	0.20710	7.0810

TABLE 12 (cont'd)
(Situation: n = 128, k = 127, r = 8, d = 1.00σ)

(e)

#	CONTRAST	#R	STD. DEV.	STANDARDIZED CONTRAST
1	0.0056	0	0.00555	0.0222
2	0.0113	0	0.00769	0.0452
3	0.0169	0	0.00952	0.0678
4	0.0227	0	0.01084	0.0908
5	0.0284	0	0.01195	0.1136
6	0.0340	0	0.01301	0.1362
7	0.0399	0	0.01397	0.1596
8	0.0457	0	0.01500	0.1826
9	0.0514	0	0.01575	0.2055
10	0.0571	0	0.01652	0.2286
11	0.0629	0	0.01719	0.2514
12	0.0687	0	0.01801	0.2750
13	0.0747	0	0.01860	0.2988
14	0.0807	0	0.01911	0.3228
15	0.0866	1	0.01975	0.3463
16	0.0927	0	0.02029	0.3707
17	0.0987	1	0.02077	0.3948
18	0.1049	1	0.02135	0.4195
19	0.1111	0	0.02196	0.4445
20	0.1174	0	0.02240	0.4697
21	0.1237	0	0.02294	0.4947
22	0.1301	0	0.02338	0.5203
23	0.1367	0	0.02395	0.5467
24	0.1433	1	0.02440	0.5732
25	0.1500	2	0.02491	0.5999
26	0.1569	0	0.02541	0.6275
27	0.1639	1	0.02600	0.6557
28	0.1710	2	0.02625	0.6841
29	0.1782	2	0.02676	0.7129
30	0.1854	2	0.02716	0.7417
31	0.1932	2	0.02766	0.7729
32	0.2009	1	0.02809	0.8035
33	0.2087	3	0.02879	0.8348
34	0.2168	1	0.02940	0.8670
35	0.2250	5	0.02987	0.9000
36	0.2336	3	0.03044	0.9344
37	0.2423	6	0.03100	0.9691
38	0.2515	8	0.03145	1.0059
39	0.2610	13	0.03216	1.0438
40	0.2709	5	0.03293	1.0835
41	0.2813	7	0.03377	1.1251
42	0.2919	16	0.03479	1.1676
43	0.3031	14	0.03557	1.2126
44	0.3150	15	0.03677	1.2600
45	0.3276	21	0.03803	1.3104
46	0.3412	33	0.03941	1.3649
47	0.3558	22	0.04061	1.4231
48	0.3717	45	0.04243	1.4867
49	0.3892	56	0.04420	1.5567
50	0.4080	82	0.04644	1.6320
51	0.4301	76	0.04917	1.7203
52	0.4554	161	0.05249	1.8217
53	0.4880	288	0.05713	1.9522
54	0.5291	498	0.06455	2.1164
55	0.5888	1037	0.07776	2.3552
56	0.6971	3405	0.10437	2.7886
57	0.7961	4416	0.11008	3.1845
58	0.8840	4824	0.10878	3.5360
59	0.9632	4949	0.10717	3.8526
60	1.0380	4985	0.10890	4.1519
61	1.1194	4991	0.11316	4.4777
62	1.2153	4999	0.12284	4.8613
63	1.3550	5000	0.15014	5.4200

TABLE 12 (cont'd)
(Situation: n = 128, k = 127, r = 8, d = 1.00σ)

(f)

#	CONTRAST	#R	STD. DEV.	STANDARDIZED CONTRAST
1	0.0055	0	0.00545	0.0222
2	0.0113	0	0.00778	0.0452
3	0.0169	0	0.00950	0.0677
4	0.0225	0	0.01090	0.0900
5	0.0282	0	0.01219	0.1127
6	0.0338	0	0.01324	0.1353
7	0.0396	0	0.01422	0.1585
8	0.0455	0	0.01516	0.1819
9	0.0513	0	0.01597	0.2052
10	0.0572	0	0.01677	0.2288
11	0.0629	0	0.01741	0.2517
12	0.0689	0	0.01802	0.2754
13	0.0747	0	0.01870	0.2987
14	0.0808	0	0.01937	0.3230
15	0.0868	0	0.02003	0.3472
16	0.0928	0	0.02045	0.3713
17	0.0989	0	0.02111	0.3957
18	0.1051	0	0.02161	0.4206
19	0.1114	0	0.02212	0.4455
20	0.1178	0	0.02263	0.4711
21	0.1242	0	0.02312	0.4969
22	0.1307	0	0.02352	0.5229
23	0.1373	0	0.02401	0.5491
24	0.1439	0	0.02449	0.5756
25	0.1506	0	0.02501	0.6023
26	0.1574	0	0.02555	0.6297
27	0.1643	0	0.02613	0.6570
28	0.1712	0	0.02661	0.6847
29	0.1782	0	0.02696	0.7130
30	0.1856	0	0.02759	0.7424
31	0.1930	0	0.02805	0.7719
32	0.2006	0	0.02840	0.8025
33	0.2086	0	0.02889	0.8345
34	0.2170	0	0.02948	0.8680
35	0.2252	0	0.03007	0.9009
36	0.2335	0	0.03075	0.9340
37	0.2423	1	0.03106	0.9694
38	0.2515	0	0.03163	1.0060
39	0.2611	0	0.03239	1.0446
40	0.2713	0	0.03317	1.0851
41	0.2814	0	0.03360	1.1256
42	0.2923	0	0.03457	1.1690
43	0.3035	2	0.03544	1.2142
44	0.3154	0	0.03654	1.2618
45	0.3278	1	0.03754	1.3111
46	0.3414	1	0.03893	1.3655
47	0.3561	1	0.04022	1.4242
48	0.3718	1	0.04169	1.4871
49	0.3896	2	0.04404	1.5582
50	0.4093	4	0.04654	1.6373
51	0.4322	4	0.04966	1.7287
52	0.4595	10	0.05410	1.8379
53	0.4929	18	0.06108	1.9714
54	0.5399	51	0.07071	2.1594
55	0.6198	236	0.09232	2.4790
56	0.9001	4698	0.13805	3.6003
57	1.0358	4975	0.12001	4.1431
58	1.1290	4997	0.11184	4.5161
59	1.2107	4998	0.10891	4.8429
60	1.2866	5000	0.10949	5.1465
61	1.3678	5000	0.11420	5.4712
62	1.4632	5000	0.12412	5.8528
63	1.6054	5000	0.15289	6.4214

TABLE 12 (cont'd)
(Situation: n = 128, k = 127, r = 8, d = 1.00σ)

(g)

#	CONTRAST	#R	STD. DEV.	STANDARDIZED CONTRAST
1	0.0057	0	0.00546	0.0226
2	0.0113	0	0.00764	0.0452
3	0.0171	0	0.00952	0.0685
4	0.0229	0	0.01084	0.0915
5	0.0287	0	0.01205	0.1150
6	0.0344	0	0.01311	0.1378
7	0.0401	0	0.01401	0.1603
8	0.0460	0	0.01502	0.1839
9	0.0518	0	0.01590	0.2072
10	0.0576	0	0.01667	0.2303
11	0.0634	0	0.01730	0.2536
12	0.0692	0	0.01803	0.2767
13	0.0751	0	0.01868	0.3005
14	0.0810	0	0.01931	0.3241
15	0.0870	0	0.01986	0.3481
16	0.0932	0	0.02034	0.3727
17	0.0992	0	0.02080	0.3968
18	0.1054	0	0.02141	0.4217
19	0.1116	0	0.02179	0.4464
20	0.1180	0	0.02239	0.4718
21	0.1244	0	0.02280	0.4975
22	0.1309	0	0.02325	0.5236
23	0.1376	0	0.02395	0.5503
24	0.1442	0	0.02458	0.5766
25	0.1509	0	0.02515	0.6035
26	0.1578	0	0.02563	0.6311
27	0.1647	0	0.02618	0.6590
28	0.1719	0	0.02668	0.6878
29	0.1790	0	0.02710	0.7160
30	0.1864	0	0.02765	0.7456
31	0.1941	0	0.02834	0.7764
32	0.2018	0	0.02900	0.8073
33	0.2096	0	0.02964	0.8385
34	0.2176	0	0.03031	0.8704
35	0.2258	0	0.03076	0.9031
36	0.2343	0	0.03132	0.9373
37	0.2431	0	0.03180	0.9723
38	0.2521	0	0.03258	1.0086
39	0.2617	0	0.03344	1.0466
40	0.2716	0	0.03422	1.0863
41	0.2818	0	0.03482	1.1271
42	0.2924	0	0.03558	1.1695
43	0.3036	0	0.03656	1.2143
44	0.3155	0	0.03774	1.2620
45	0.3285	0	0.03887	1.3139
46	0.3421	0	0.03991	1.3683
47	0.3566	0	0.04138	1.4266
48	0.3727	0	0.04321	1.4909
49	0.3903	0	0.04508	1.5611
50	0.4100	0	0.04699	1.6399
51	0.4322	0	0.05002	1.7288
52	0.4594	0	0.05419	1.8377
53	0.4938	0	0.06074	1.9752
54	0.5408	3	0.07134	2.1630
55	0.6235	21	0.09662	2.4938
56	1.1429	4977	0.15192	4.5717
57	1.2859	4999	0.12378	5.1437
58	1.3813	5000	0.11336	5.5253
59	1.4619	5000	0.10999	5.8477
60	1.5382	5000	0.11083	6.1528
61	1.6200	5000	0.11246	6.4800
62	1.7159	5000	0.12419	6.8635
63	1.8597	5000	0.15253	7.4387

TABLE 12 (cont'd)
(Situation: n = 128, k = 127, r = 8, d = 1.00σ)

(h)

#	CONTRAST	#R	STD. DEV.	STANDARDIZED CONTRAST
1	0.0019	0	0.00188	0.0107
2	0.0038	0	0.00258	0.0213
3	0.0056	0	0.00317	0.0319
4	0.0075	0	0.00365	0.0425
5	0.0093	0	0.00402	0.0529
6	0.0112	0	0.00440	0.0636
7	0.0132	0	0.00478	0.0745
8	0.0151	0	0.00508	0.0851
9	0.0170	0	0.00540	0.0959
10	0.0188	0	0.00562	0.1065
11	0.0207	0	0.00587	0.1170
12	0.0225	0	0.00612	0.1273
13	0.0244	0	0.00632	0.1381
14	0.0263	0	0.00652	0.1487
15	0.0282	0	0.00674	0.1594
16	0.0301	0	0.00697	0.1703
17	0.0319	0	0.00716	0.1807
18	0.0339	0	0.00734	0.1916
19	0.0358	0	0.00754	0.2023
20	0.0377	0	0.00770	0.2133
21	0.0396	0	0.00789	0.2242
22	0.0416	0	0.00805	0.2351
23	0.0435	0	0.00819	0.2458
24	0.0454	0	0.00838	0.2566
25	0.0473	0	0.00854	0.2677
26	0.0492	0	0.00866	0.2785
27	0.0512	0	0.00882	0.2899
28	0.0532	0	0.00897	0.3009
29	0.0551	0	0.00914	0.3118
30	0.0571	0	0.00930	0.3228
31	0.0590	0	0.00943	0.3339
32	0.0610	0	0.00956	0.3451
33	0.0629	0	0.00969	0.3560
34	0.0650	0	0.00981	0.3674
35	0.0669	0	0.00993	0.3786
36	0.0690	0	0.01006	0.3900
37	0.0710	0	0.01010	0.4014
38	0.0730	0	0.01023	0.4130
39	0.0751	0	0.01032	0.4249
40	0.0771	0	0.01040	0.4362
41	0.0792	0	0.01054	0.4477
42	0.0812	0	0.01065	0.4594
43	0.0833	0	0.01073	0.4710
44	0.0854	0	0.01087	0.4831
45	0.0875	0	0.01102	0.4950
46	0.0896	0	0.01113	0.5069
47	0.0917	0	0.01129	0.5189
48	0.0939	0	0.01139	0.5311
49	0.0960	0	0.01153	0.5432
50	0.0982	0	0.01161	0.5553
51	0.1004	0	0.01173	0.5680
52	0.1026	0	0.01186	0.5806
53	0.1049	0	0.01196	0.5932
54	0.1071	0	0.01205	0.6058
55	0.1093	0	0.01220	0.6182
56	0.1115	0	0.01228	0.6308
57	0.1138	0	0.01241	0.6437
58	0.1161	0	0.01250	0.6568
59	0.1184	0	0.01260	0.6699
60	0.1207	0	0.01272	0.6830
61	0.1231	0	0.01282	0.6964
62	0.1255	0	0.01297	0.7097
63	0.1278	0	0.01308	0.7232
64	0.1303	0	0.01324	0.7373
65	0.1329	0	0.01333	0.7515
66	0.1354	0	0.01340	0.7657
67	0.1378	0	0.01359	0.7798
68	0.1403	0	0.01371	0.7939
69	0.1429	0	0.01385	0.8081
70	0.1454	0	0.01402	0.8227
71	0.1481	0	0.01414	0.8377
72	0.1507	0	0.01427	0.8526
73	0.1534	0	0.01440	0.8677
74	0.1561	0	0.01449	0.8833
75	0.1589	0	0.01460	0.8990
76	0.1616	0	0.01468	0.9144
77	0.1644	0	0.01483	0.9302
78	0.1673	0	0.01493	0.9463
79	0.1702	0	0.01511	0.9629
80	0.1732	0	0.01523	0.9799
81	0.1762	0	0.01539	0.9966
82	0.1792	0	0.01558	1.0140
83	0.1823	0	0.01562	1.0313
84	0.1855	0	0.01575	1.0494
85	0.1888	0	0.01599	1.0679
86	0.1921	0	0.01615	1.0869
87	0.1955	0	0.01629	1.1060
88	0.1989	0	0.01650	1.1250
89	0.2024	0	0.01673	1.1450
90	0.2060	0	0.01690	1.1654
91	0.2097	0	0.01709	1.1860
92	0.2135	0	0.01732	1.2077
93	0.2172	0	0.01765	1.2288
94	0.2212	0	0.01790	1.2512
95	0.2253	0	0.01811	1.2742
96	0.2295	0	0.01832	1.2980
97	0.2338	0	0.01859	1.3227
98	0.2382	0	0.01880	1.3477
99	0.2429	0	0.01926	1.3740
100	0.2476	0	0.01961	1.4005
101	0.2524	0	0.02008	1.4279
102	0.2575	0	0.02042	1.4569
103	0.2629	0	0.02079	1.4873
104	0.2685	0	0.02118	1.5189
105	0.2744	0	0.02159	1.5524
106	0.2806	0	0.02215	1.5875
107	0.2872	0	0.02248	1.6246
108	0.2941	0	0.02323	1.6638
109	0.3015	0	0.02408	1.7058
110	0.3096	1	0.02495	1.7514
111	0.3182	0	0.02592	1.8002
112	0.3277	0	0.02672	1.8537
113	0.3382	1	0.02797	1.9133
114	0.3504	0	0.02956	1.9822
115	0.3643	2	0.03165	2.0609
116	0.3808	2	0.03431	2.1541
117	0.4014	10	0.03794	2.2707
118	0.4313	16	0.04530	2.4396
119	0.4833	86	0.06184	2.7337
120	0.7493	4889	0.10396	4.2385
121	0.8501	4993	0.08733	4.8090
122	0.9183	5000	0.08054	5.1945
123	0.9753	5000	0.07752	5.5170
124	1.0297	5000	0.07798	5.8249
125	1.0855	5000	0.08081	6.1406
126	1.1529	5000	0.08759	6.5216
127	1.2552	5000	0.10822	7.1002

TABLE 13. COMPARING R-SPILLOVER OF NORMAL AND HALF-NORMAL PLOTS
(5000 runs, Population sigma = 1.00)

$d_s =$	$+1.00\sigma$	$+1.25\sigma$	$+1.50\sigma$	$+2.00\sigma$
N = 32, K = 31, r = 8, 1st error rank = 23				
Normal	33%(20)*	23%(22)	10%(23)	0.3%(23)
Half-normal	32%(19)	25%(21)	12%(22)	0.5%(23)
N = 64, K = 63, r = 8, 1st error rank = 55				
Normal	18%(54)	4%(55)	0.3%(55)	
Half-normal	20%(54)	5%(55)	0.4%(55)	
N = 128, K = 127, r = 8, 1st error rank = 119				
Normal	2%(119)			
Half-normal	2%(119)			

* The number in parentheses is the first rank with less than 10% R-spillover.

For all practical purposes, the degree of R-spillover for the aggregate normal and half-normal plot data are not considerably different. While the normal plot data might appear to have a slight overall edge, having a slightly smaller R-spillover into the intended error ranks and having fewer ranks to inspect for real effects that have at least a 10% chance of being there, these differences are likely to be obscured in a single experiment. For the aggregate R-spillover data, a difference of 1% or 2% is probably within the accuracy of the data.

With normal plot data, we have seen how the R-spillover for the same number of real effects is less at the borders between intended real and error ranks when there are both positive and negative real effects, than is at the border when all are positive. Therefore, for the same number of real effects, the normal situation would be expected to result in a smaller R-spillover than the half-normal when positive and negative effects are present. This means that the chance of selecting error contrasts decreases with the normal plots.

ESTIMATING SIGMA. To decide whether an effect is real or not with a certain error rate, the population sigma must be known. The population sigma can be estimated by calculating the slope of the standardized contrasts. These requirements lead to two circular situations, each a classic "Catch 22."

1. The exact number of the errors contrast can be known by identifying which contrasts represent real effects; the remaining ones, of course, are the error contrasts. But to determine how many contrasts are real, one must have an estimate of the standard error of the contrasts, which is derived from knowledge of the slope of the error contrasts. The accuracy of this estimate decreases markedly when the exact number of error contrasts are not known.

2. Standardized values must be used when the slope of the error contrasts used to estimate the population sigma. To standardize the error contrasts, the correct estimate of the population sigma is required. While other ways have been suggested for estimating sigma, the slope of the contrasts is still the most accurate.

As the size of the real effects diminish or the number of real effects increase, the R-spillover also increases and the chance of determining the exact number of real effects and error contrasts also decreases.

With normal plots, there is an additional problem. We must not only decide how many error contrasts there are but also at what ranks they are located. Failure in either case means that we are plotting the error contrasts against the incorrect estimated values of order statistics. This result is an incorrect slope.

OVER- AND UNDERESTIMATING THE NUMBER OF ERROR CONTRASTS. Let us determine what happens to our estimate of the population sigma if we over- or underestimate the number of error contrasts in normal and half-normal plot data. The following situation will be used:

$$n = 32,\ k = 31,\ r = 8,\ d_e = +1.00 \qquad [e = k - r = 23]$$

As Zahn suggested, only the smallest 0.70 of the contrasts assumed to be error are used to calculate the slope. The results are given in Table 13.

TABLE 14. COMPARING THE ROBUSTNESS OF NORMAL AND HALF-NORMAL PLOTS ON POPULATION SIGMA ESTIMATIONS WHEN THE NUMBER OF ERROR CONTRASTS ARE OVER- AND UNDERESTIMATED

Number of Contrasts Used (0.7e)	Population Estimates Mean Slope (Normal)		Mean Slope (Half-normal)
18 out of 26 (More)	0.95s	[Aver. 1.05s]	1.15s
18 out of 25			1.09s
17 out of 24			1.04s
16 out of 23 (TRUE)	1.00s	[Aver. 1.00s]	1.00s
16 out of 22			0.93s
15 out of 21			0.88s
14 out of 20 (Less)	1.14s	[Aver. .99s]	0.84s

Note the inverse relationships that exist between the normal and the half-normal plot estimates when too many or too few error contrasts are used to estimate the sigma. For the normal plot data, underestimating the number of error contrasts lead to an overestimation of sigma, while with the half-normal plot data, it leads to an underestimation. The data in Table 14 suggests that estimates of the population sigma from normal plot data is somewhat more robust than from half-normal plot data, at least when the number of error contrasts are overestimated.

Since the data can be plotted either way once it has been obtained, the reciprocal results in Table 14 suggest that the best estimate of sigma can be obtained, whether the number of error contrasts are over- or underestimated, by averaging the estimate from the normal order data with that from the half-normal data. In the above table, when this was done with the correct answer being 1.00, we obtained an average value of 1.05 when 18 out of 26 error contrasts were used to calculate the slope and 0.99 when 14 out of 20 were used.

WHAT PROPORTION OF THE ERROR CONTRASTS SHOULD BE USED? In Version 5, Zahn threw away the larger 0.30 of the error contrasts before calculating the slope of the standardized contrasts. By noting how R-spillover relates to the degradation in the error contrasts from the expected values, it is apparent that this rule, empirically determined, is a consequence of the R-spillover that occurred. More R-spillover would be expected to occur in data from a n = 16 experiment than might be expected with larger experiments. In the half-normal plot, with four real effects out of 15 contrasts, Zahn's typical situation, the three largest contrasts -- 0.30 of the 11 error contrasts -- would not be included in the calculation of his slope. Those contrasts fall at the ranks where the greatest amount of spillover would ordinarily occur for moderate-sized effects. However, applying the same proportion to a larger design, e.g., n = 32, k = 31, with four real effects, the eight largest error contrasts would not be used out of 27. In view of the data shown in Table 12, that could be overly conservative.

SINGLE RUN DATA

Up to this point, we have been examining *aggregate* data, the mean of 5000 runs. This tends to provide a cleaner picture than one would expect to find in a real-world, single-run experiment. In the remainder of this section, we will look at some results from individual runs, results taken at random from the 5000 runs of the earlier analyses.

DETECTION. In Figures 5 through 8, each normal plot represents a single run that was purposefully selected -- to illustrate a particular point -- from a set of 50 which the computer had randomally selected from the complete set of 5000 runs. The situations from which these plots were selected are:

Figure 5: $n = 32,\ k = 31,\ r = 8+,\ d = 1.00\sigma$

Figure 6: $n = 32,\ k = 31,\ r = 8+,\ d = 1.25\sigma$

Figure 7: $n = 32,\ k = 31,\ r = 5+,3-,\ d = +1.25\sigma$

Figure 8: $n = 32,\ k = 31,\ r = 8+,\ d = +1.67\sigma$

In both Figures 5-a and 5-b, the effects in the first seven ranks appear sufficiently off the error line to be considered real. However, this is incorrect. For this set of data, the actual locations of the real effects had been tracked. In fact, in Figure 5-a, the plot seventh from the largest was actually an error contrast and the plots in the eighth and seventeenth ranks were the remaining two real effects (R-spillover). In Figure 5-b, however, there were no inversions of real and error contrasts; the contrast in the eighth rank is actually real, although this would not have been detected by observing these plots. In Figure 5-c, there appear to be nine real effects, although in fact there are only eight; the contrast in the sixth rank is actually an error contrast (E-spillover). In Figure 5-d, it is almost impossible to determine how many real effects might be present. Actually, the eighth rank is an error contrast and the ninth rank is the last real effect.

In Figure 6, with an effect of more moderate size, $d = 1.25\sigma$, we see a plot (Figure 6-a) in which all eight might be detected, although the transition from real to error is not sharp (which could conceivably be the result of an inversion of real and error effects). In Figure 6-b, only six of the eight real effects are distinguishable. In Figure 6-c, a ninth additional real positive effect apparently stands out and there might be one real negative effect. Since the tracking capability had not been written into the simulation program when these plots were generated, we have no idea to what extent these apparent interpretations are correct.

In Figure 7, we also have eight real effects, $d = 1.25$, but five of them are positive and three negative. How does this show up on the plots? With normal plots and with the e.v.n.o.s. on the ordinate, the positive real effects should fall off below the line and the negative real effects should fall off above the line. Falling on the opposite sides indicates that the data may be truncated. From the plots in this figure, it can be seen that sometimes they're all apparently clearly visible (Figure 7-a), and sometimes none of them are (Figure 7-b). Much of the time, detectability falls in between, for example in Figure 7-c, with the five positive ones being questionable, and the three negatives being more clearly visible.

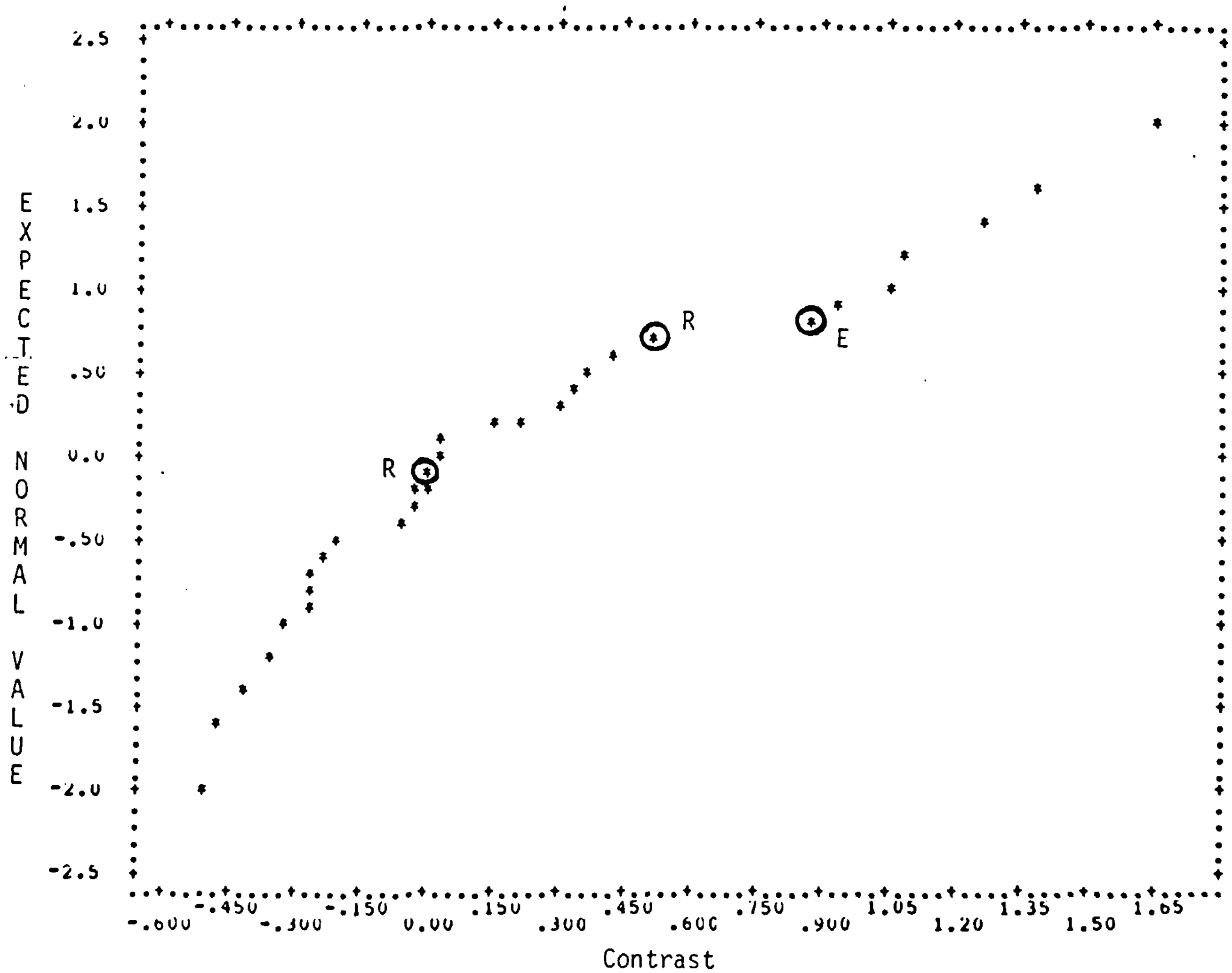


Figure 5-a. Sample Individual Normal Plot for Situation: n=32, k=31, r=8+, d = 1.00σ

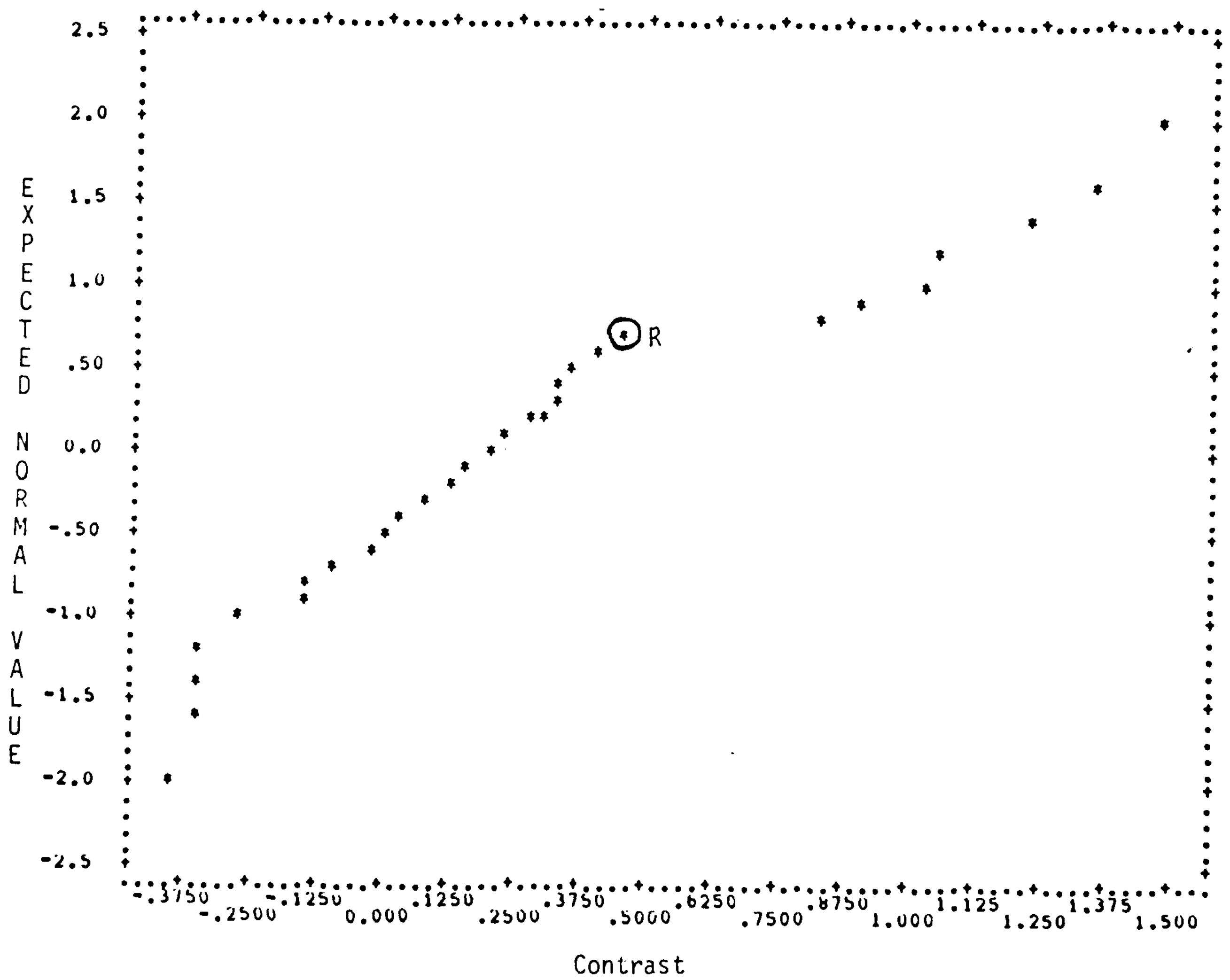


Figure 5-b. Sample Individual Normal Plot for Situation: n=32, k=31, r=8+, d = 1.00σ

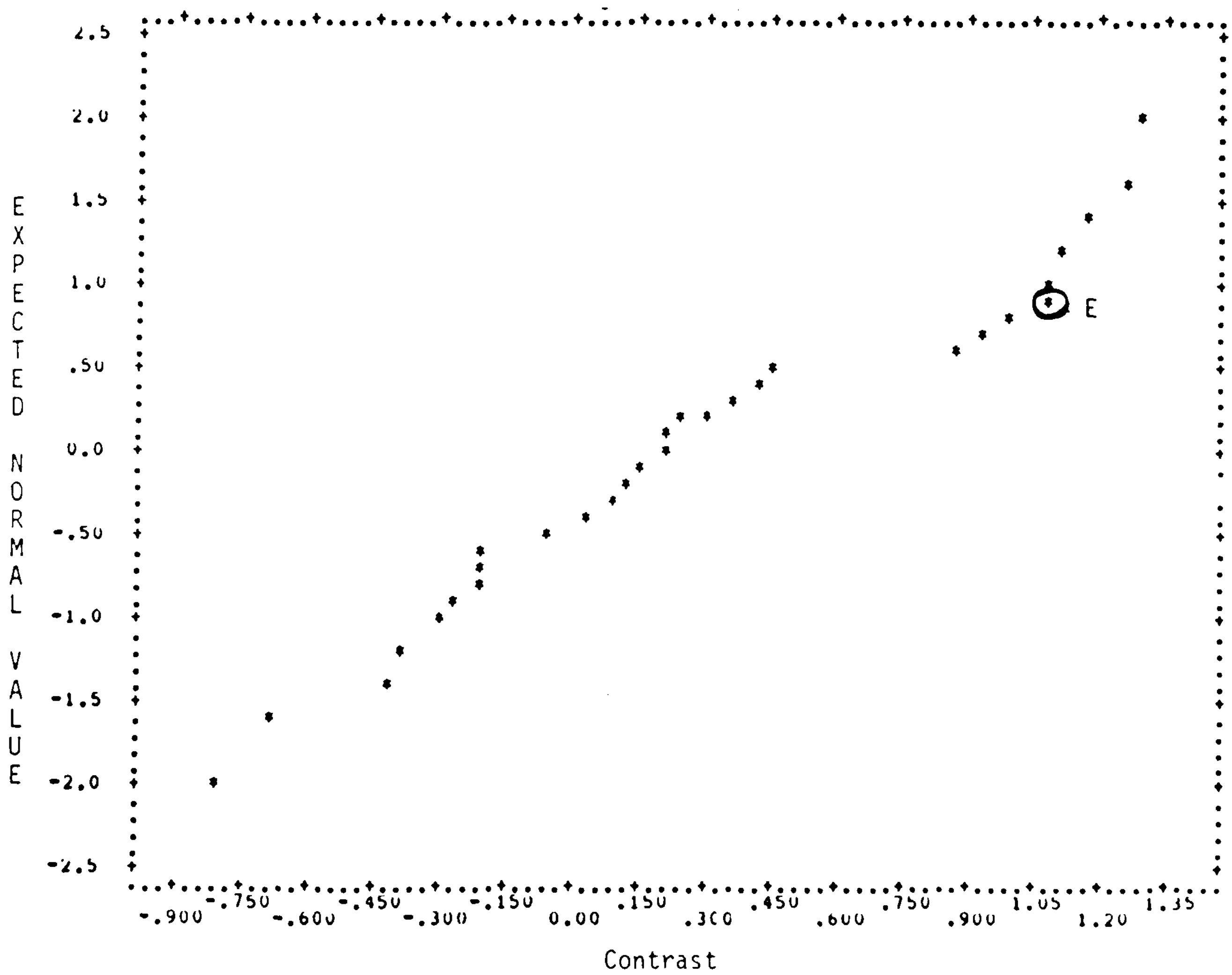


Figure 5-c. Sample Individual Normal Plot for Situation:
n=32, k=31, r=8+, d = 1.00σ

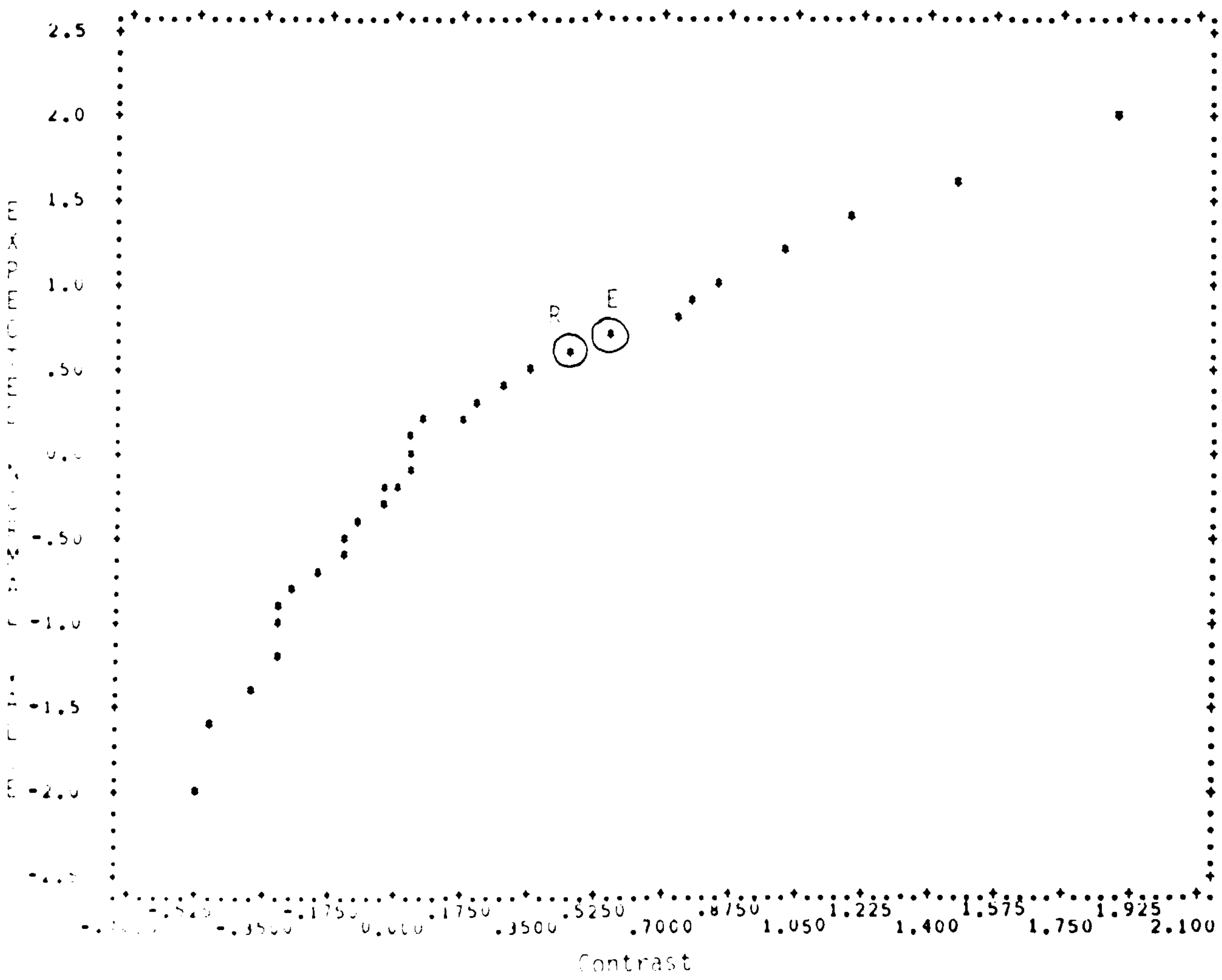


Figure [illegible]. Sample Individual Normal Plot for Situation:
n [illegible], k 31, r 8+, d = 1.00σ

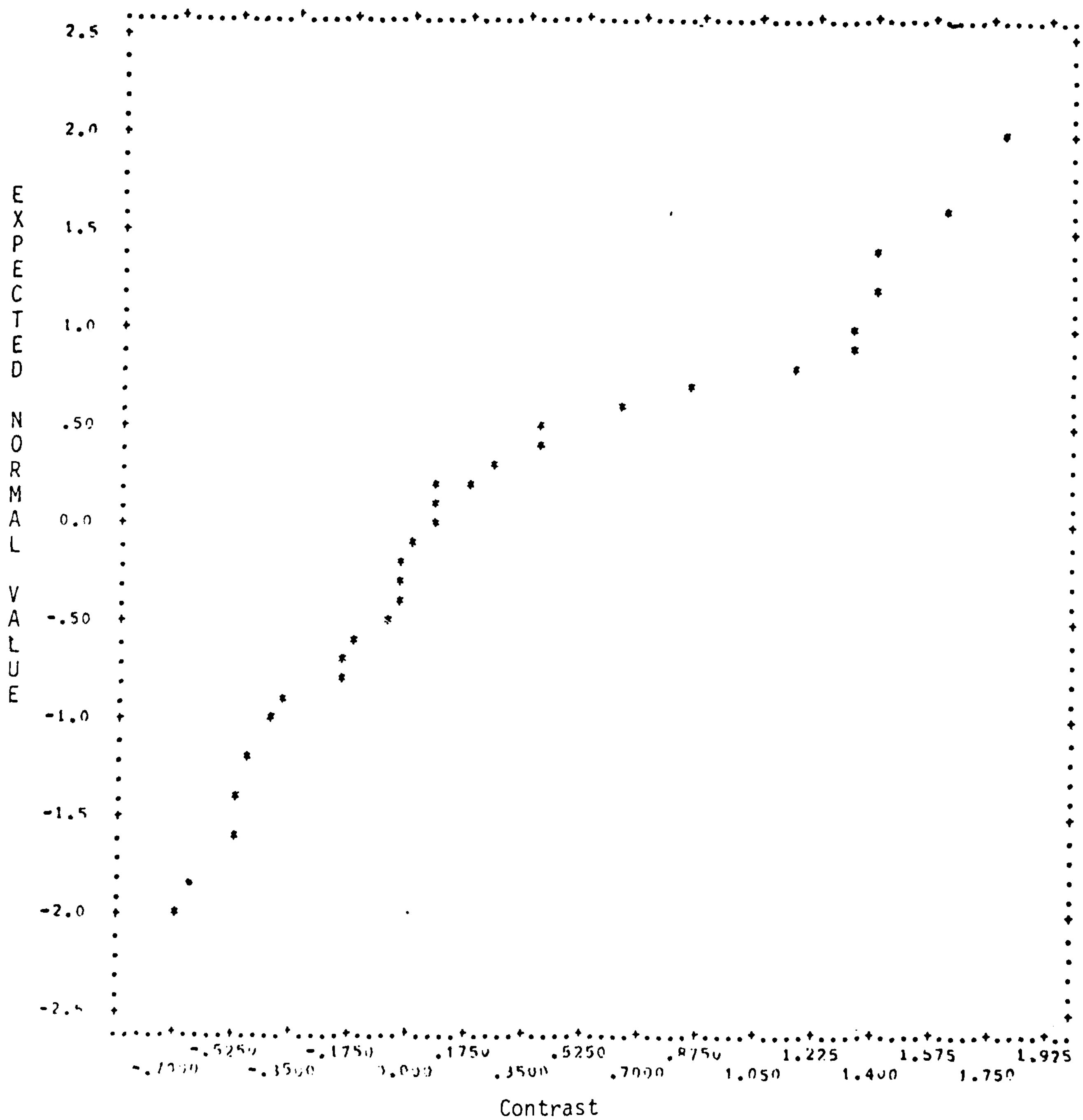


Figure 6-a. Sample Individual Normal Plot for Situation: n=32, k=31, r=8+, d = 1.25σ

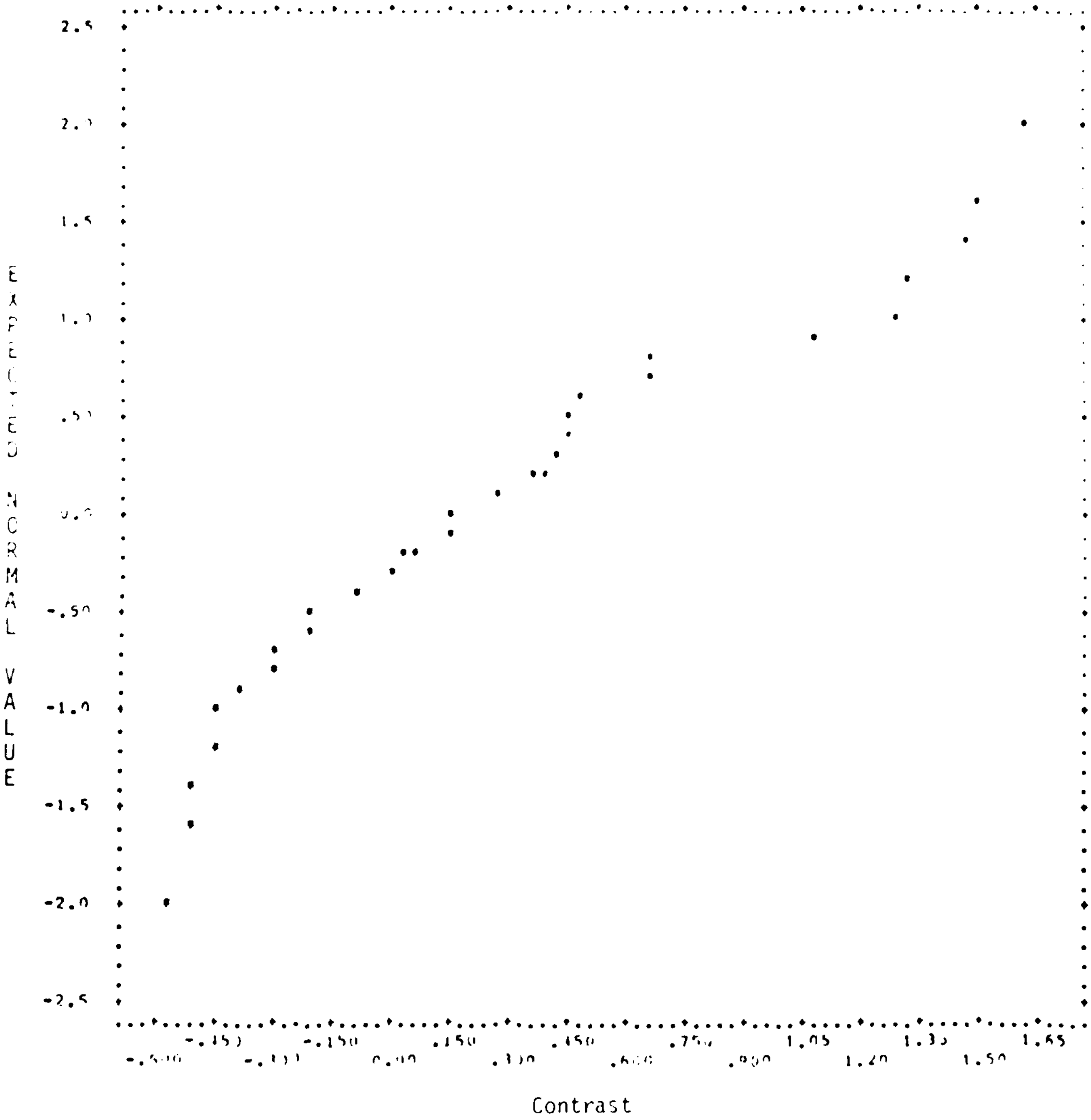


Figure 6-b. Sample Individual Normal Plot for Situation:
n=32, k=31, r=8+, d = 1.25σ

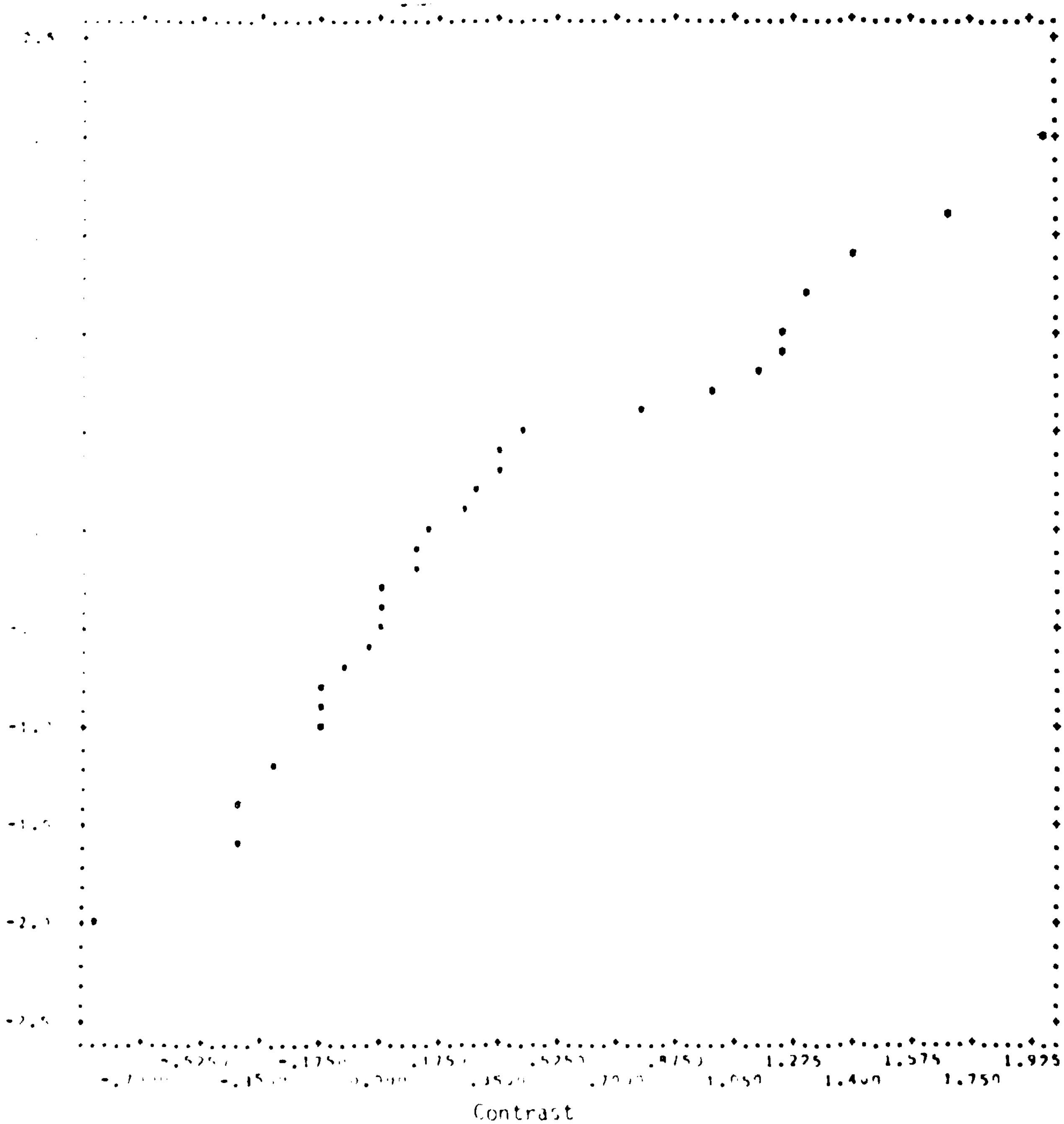


Figure 6 c. Sample Individual Normal Plot for Situation:
n=32, k=31, r=8+, d = 1.25σ

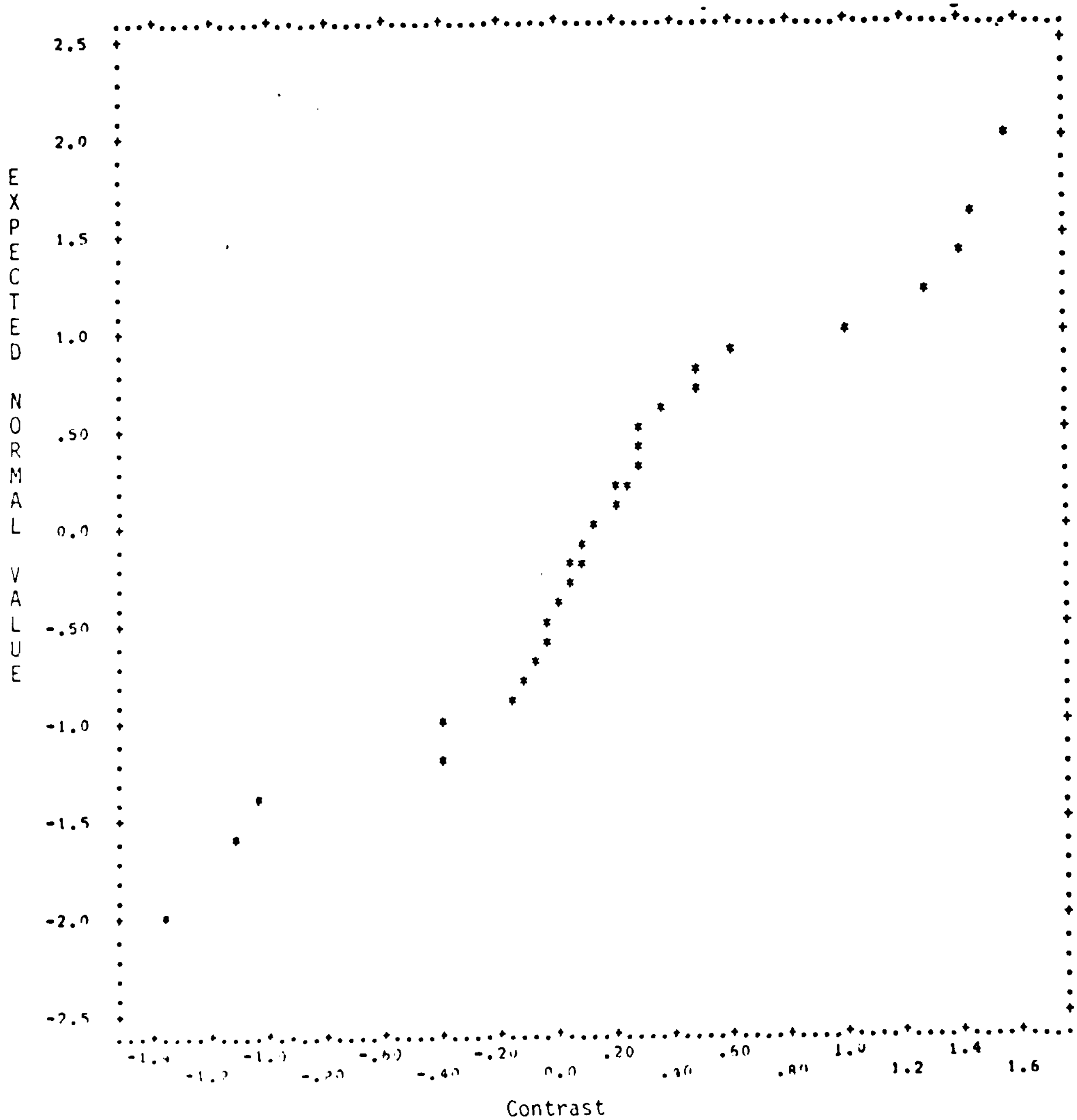


Figure 7-a. Sample Individual Normal Plot for Situation:
n=32, k=31, r=5+,3-, d = 1.25σ

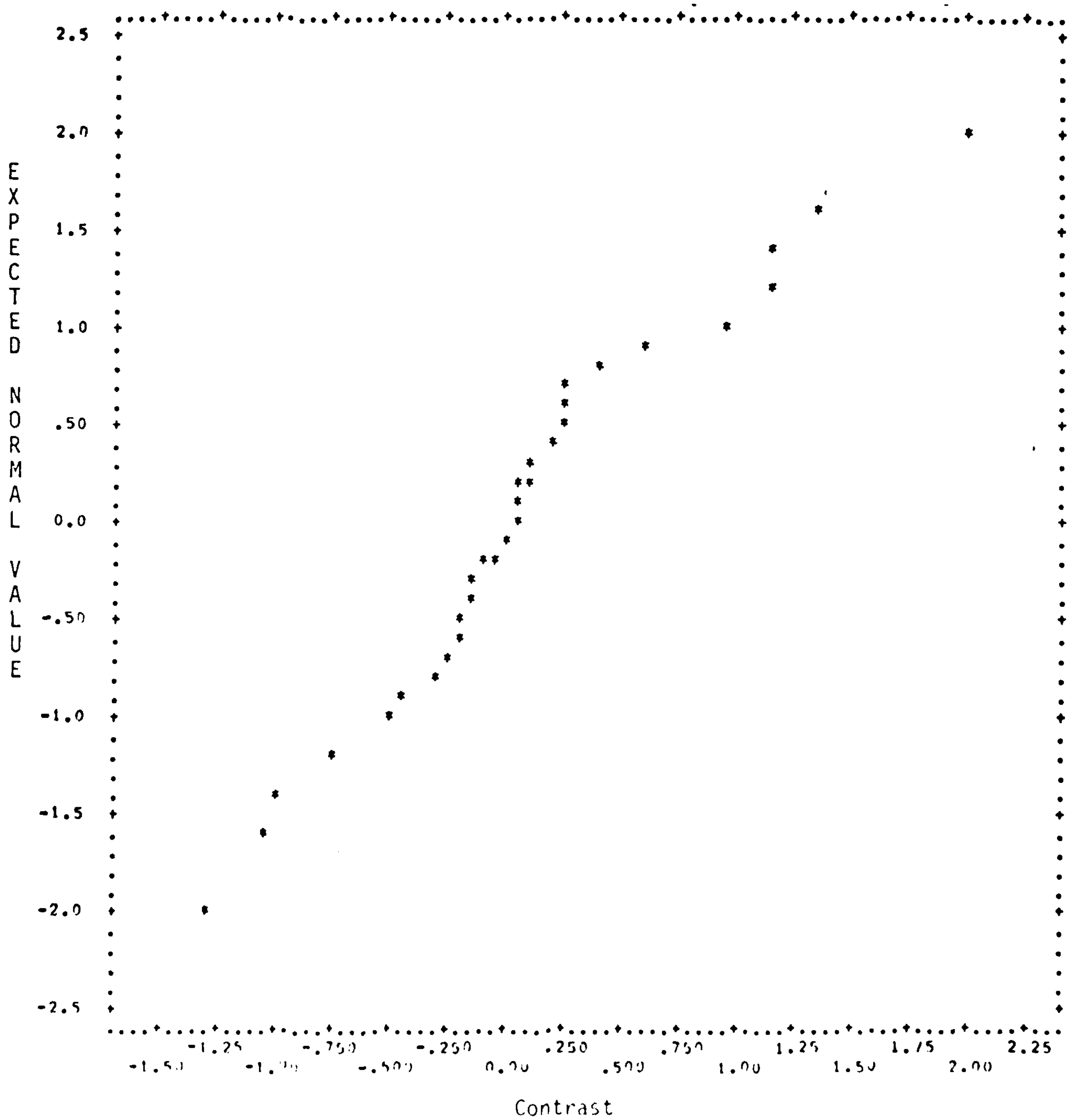


Figure 7 b. Sample Individual Normal Plot for Situation:
n=32, k=31, r=5+,3-, d = 1.25σ

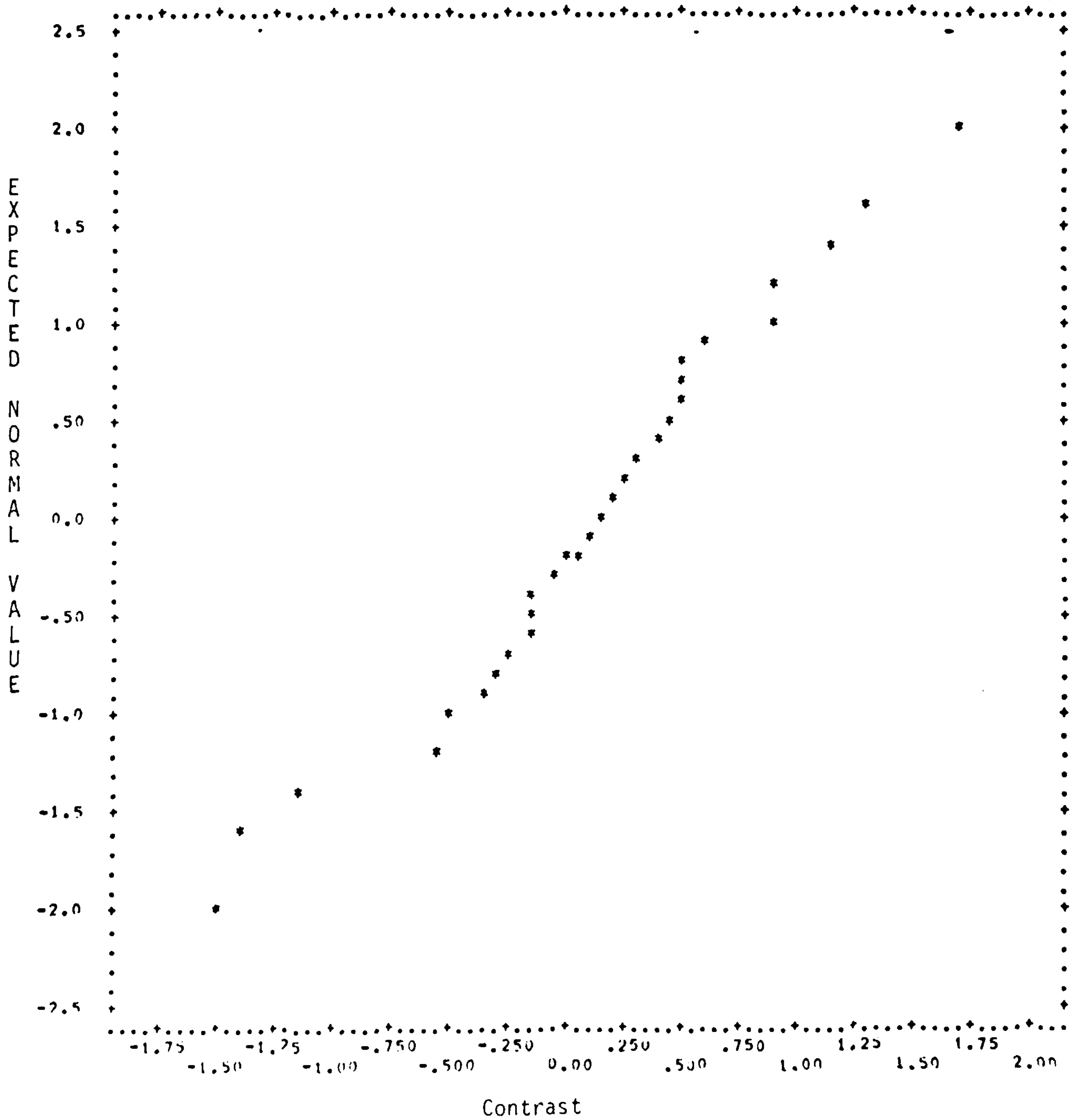


Figure 7-c. Sample Individual Normal Plot for Situation:
n=32, k=31, r=5+,3-, d = 1.25σ

Finally, in Figure 8, we can see how relatively large real effects influence individual plots. This still does not guarantee that there will not be some plots that are so ambiguous that not one of the eight effects stands out clearly (Figure 8-a). On the other hand, they may stand out quite clearly (Figure 8-b), or they may vaguely stand out, but with a slow transition rather than a distinct break between real and error contrasts (Figure 8-c). This last example makes one suspect there will be some inversions between real and error contrasts.

ESTIMATING SIGMA. In the aggregate data, estimates of the population sigma from slope calculations frequently produced satisfactory results. But those were based on the means of 5000 runs. How good might these estimates be for a single run?

This situation was investigated:

$n = 32,\ k = 31,\ r = 8,\ r+ = 8,\ d = 1.25\sigma.$

With eight real effects, there are 23 error contrasts. For this analysis, all error contrasts were at the minus end of the continuum of a normal plot. A slope was calculated for the 15 smallest standardized error contrasts out of the 23 against their corresponding ranks of the e.v.n.o.s. for k = 23. These 15 approximate the 0.70 part of the error contrasts that Zahn proposed should be used. The standardized error contrasts were used for the calculation since this enabled the slope to be the appropriate estimate of the population sigma, in this case equal to 1.00.

In Figure 9, a histogram is shown for 50 slopes (each from an individual run), selected at random from 5000 runs on the above situation. The mean slope of all 50 slopes is 1.002, which is equivalent to the true population sigma of 1.00. The standard deviation of these 50 slopes is 0.205. That means that if the distribution is essentially normal, 68% of the slopes for a single run would vary between 0.80 to 1.20. In the distribution shown in Figure 11, the values are actually 0.78 and 1.21 respectively, and the full range of individual sigmas for the same situation ranged from 0.52 to 1.45.

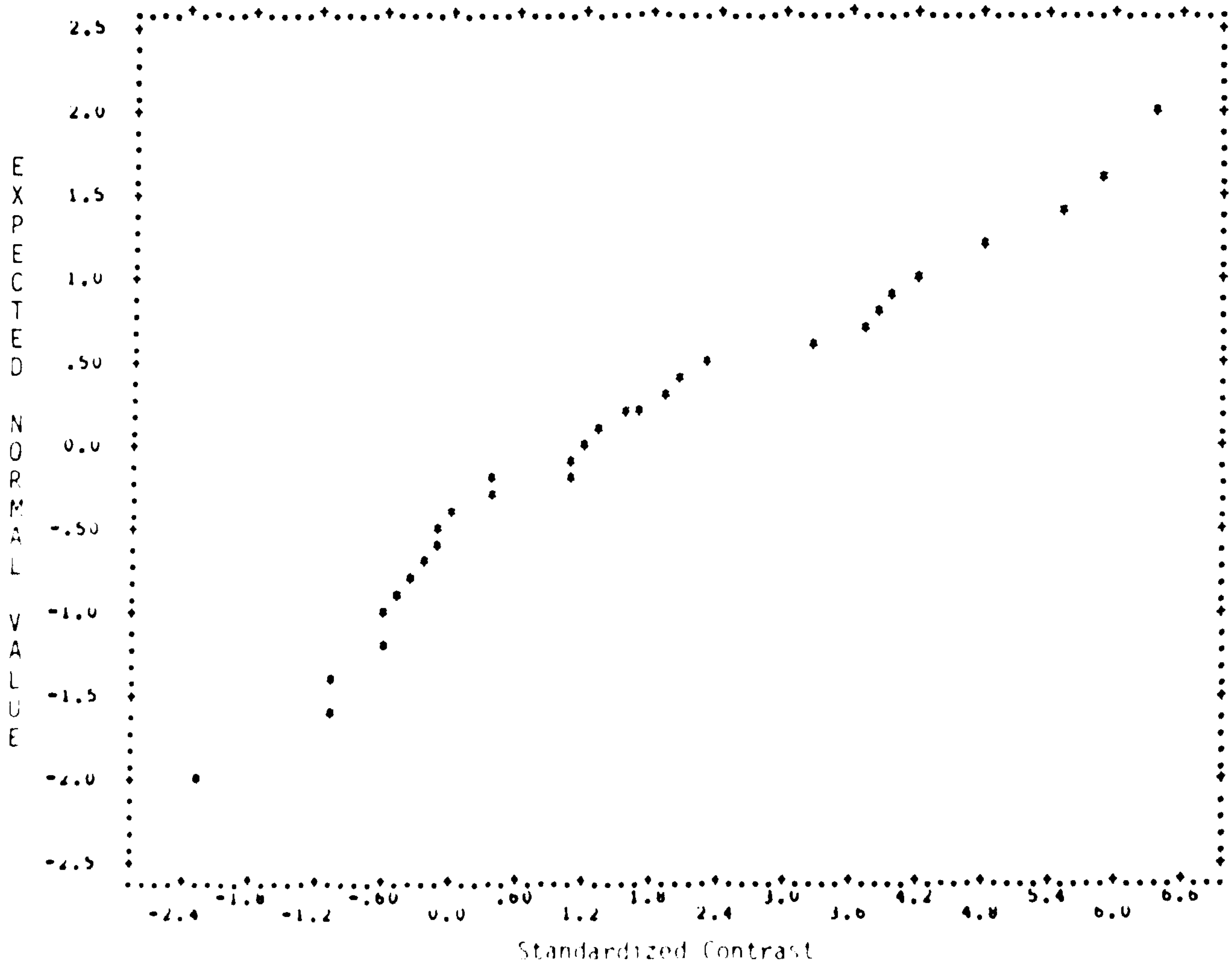


Figure 8.a. Sample Individual Plots for Situation:
[illegible] k [illegible], r .8[illegible], d 1.67[illegible]

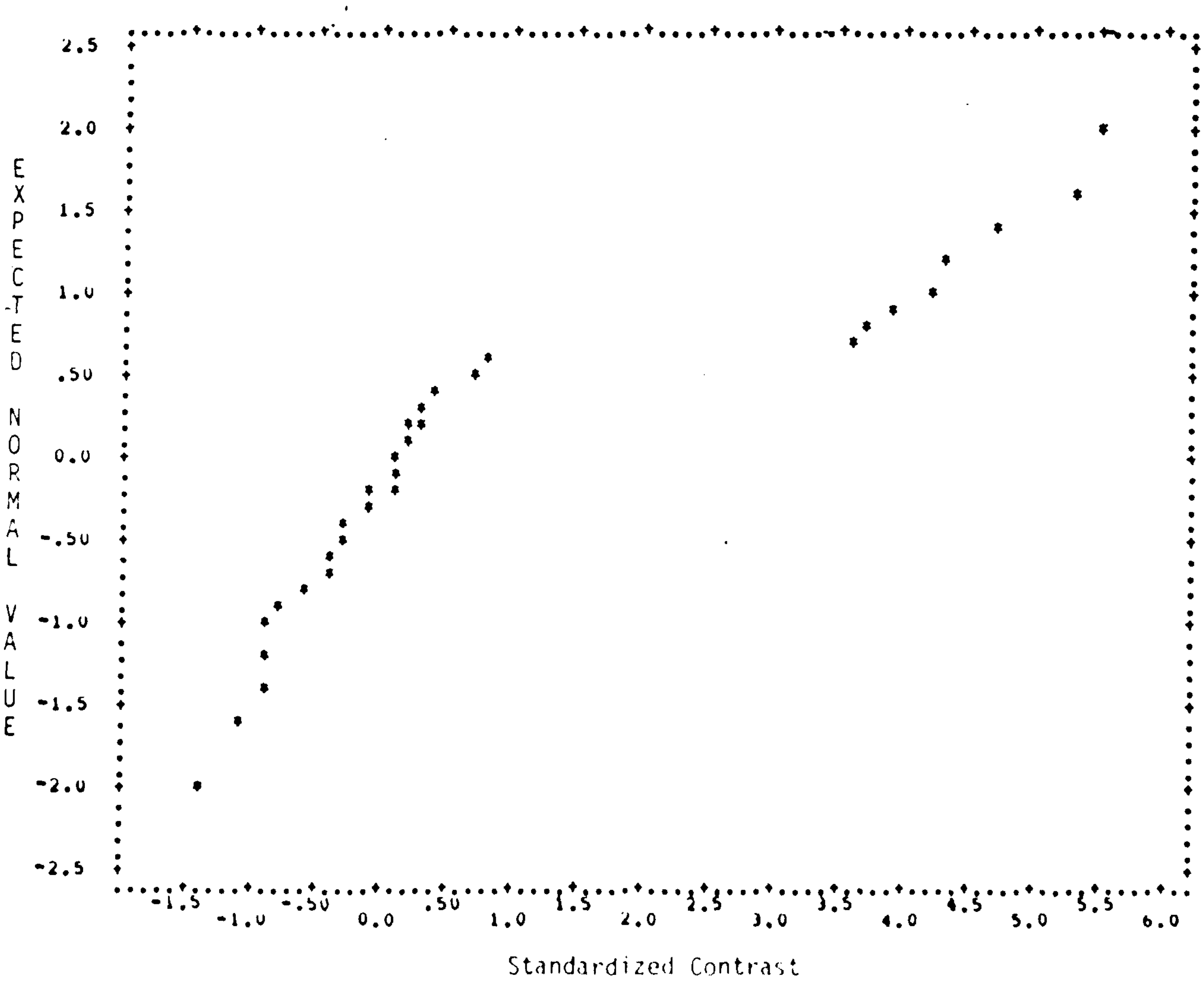
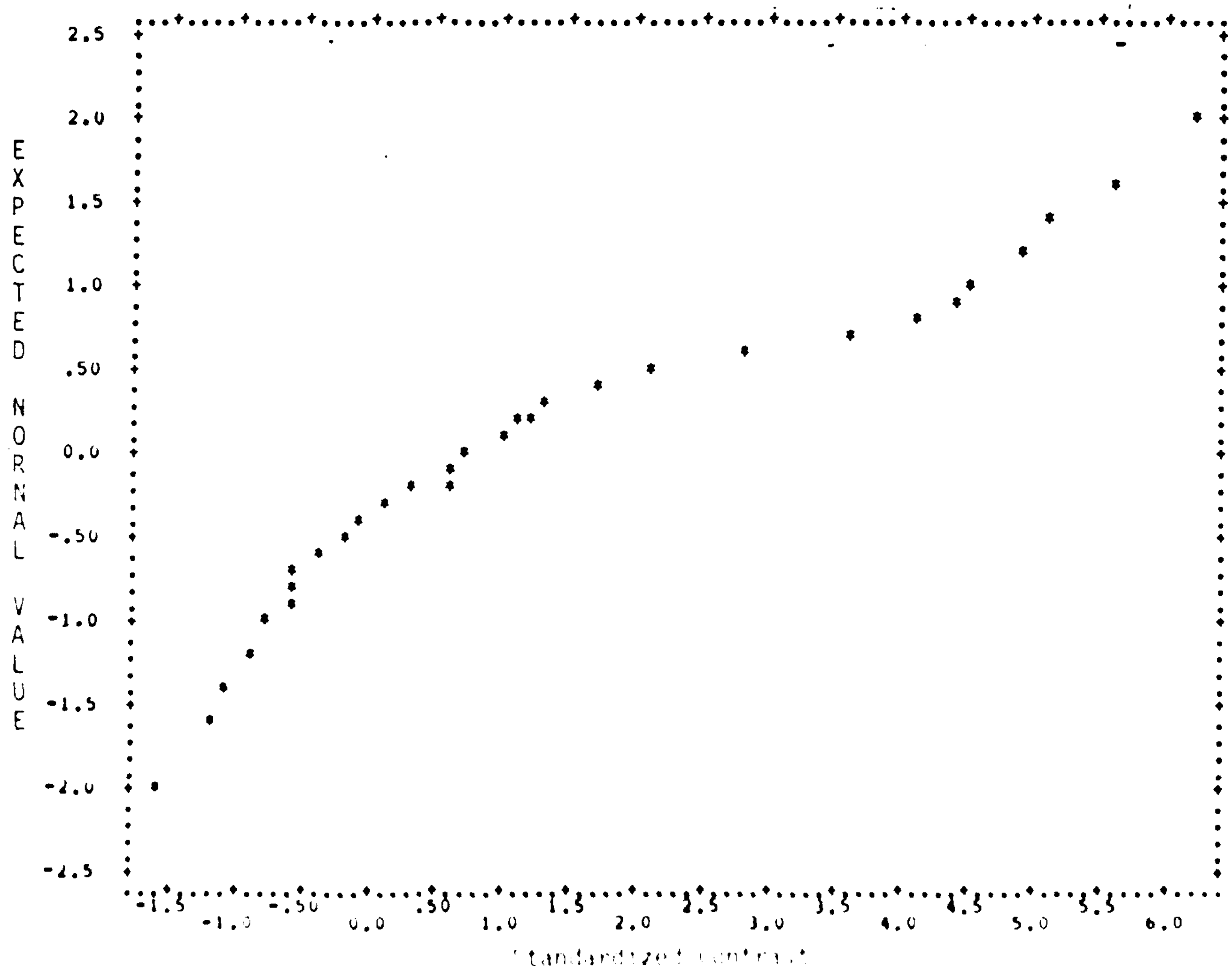


Figure 8 b. Individual Plots for Situation:
n 32, k 31, r 8+, d 1.67σ



EXPECTED NORMAL VALUE
2.5
2.0
1.5
1.0
.50
0.0
-.50
-1.0
-1.5
-2.0
-2.5
-1.5
-1.0
-.50
0.0
.50
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0

Interval Name		Frequency Int.	Frequency Cum.	Percentage Int.	Percentage Cum.
.525000	XX	2	2	4.0	4.0
.550000		0	2	0.0	4.0
.575000		0	2	0.0	4.0
.600000		0	2	0.0	4.0
.625000		0	2	0.0	4.0
.650000		0	2	0.0	4.0
.675000		0	2	0.0	4.0
.700000	X	1	3	2.0	6.0
.725000	X	1	4	2.0	8.0
.750000	X	1	5	2.0	10.0
.775000	X	1	6	2.0	12.0
.800000	XXX	3	9	6.0	18.0
.825000	X	1	10	2.0	20.0
.850000	XX	2	12	4.0	24.0
.875000	X	1	13	2.0	26.0
.900000	X	1	14	2.0	28.0
.925000	X	1	15	2.0	30.0
.950000	XX	2	17	4.0	34.0
.975000	XXXX	4	21	8.0	42.0
1.000000	XXXX	4	25	8.0	50.0
1.025000	XXX	3	28	6.0	56.0
1.050000	XX	2	30	4.0	60.0
1.075000		0	30	0.0	60.0
1.100000	XXXX	4	34	8.0	68.0
1.125000	XXXX	4	38	8.0	76.0
1.150000	X	1	39	2.0	78.0
1.175000		0	39	0.0	78.0
1.200000	X	1	40	2.0	80.0
1.225000	XXXX	4	44	8.0	88.0
1.250000		0	44	0.0	88.0
1.275000		0	44	0.0	88.0
1.300000	XX	2	46	4.0	92.0
1.325000	X	1	47	2.0	94.0
1.350000	XX	2	49	4.0	98.0
1.375000		0	49	0.0	98.0
1.400000		0	49	0.0	98.0
1.425000		0	49	0.0	98.0
1.4[illegible]	X	1	50	2.0	100.0

Figure 4. Histogram of 50 Population Sigmas
[illegible]

This wide distribution is not untypical of the other distributions that were calculated but not shown here. What is important to note is that in a single experiment, the estimated population sigma may be off more than 20% over 32% of the time strictly by chance.

The analysis of the above data, as is the case with so much of the data in this report, took advantage of the fact that we knew what the correct sigma was and how many real and error effects there really were. We estimated the standardized contrasts using the correct value for the square root of four over n, or in this case, 0.3536. But, in the real world, we don't have this perfect knowledge. Just what effect does being off on the initial estimate of sigma, used to standardize our contrasts, really make in the final estimate of the population sigma?

In Table 15, the mean slope (or final estimate of the population sigma) for 5000 runs is shown along with its standard deviation when the estimated sigma used to calculate the standardized values is over- and underestimated.

The situation used was:

$n = 32$, $k = 31$, $r = 8$, $d_{\bullet} = +1.25$ (see Table 5 c)

TABLE 15. EFFECT OF ERRORS IN THE INITIAL ESTIMATE OF SIGMA ON THE FINAL ESTIMATE OF SIGMA (50 runs)

Initial Estimate of Sigma (True Sigma = 1.00)	Final Mean Sigma	Inter 95%
[illegible]	[illegible]	[illegible] 0.66
[illegible]	[illegible]	
[illegible]	[illegible]	1.4[illegible] [illegible]
[illegible]	[illegible]	
[illegible]	[illegible]	
[illegible]	[illegible]	[illegible]

Overestimates of the initial sigma of the normal plots led to an underestimation of the population sigma, on average, and underestimation in the initial stage, to an overestimation of the population sigma. This phenomenon is strictly an arithmetic consequence of the fact that to convert the raw contrast to the standardized values used to estimate the slope, we divided by the initial estimate of sigma multiplied by the square root of four over n (see Equation 6). This can be illustrated in the following manner, holding the square root of four over n constant:

Initial Sigma	Raw Contrasts	Standardized Contrasts	Slope
1 (True)	2,4,6,8,10	2,4,6,8,10	2
2 (Over)	2,4,6,8,10	1,2,3,4,5	1
0.5 (Under)	2,4,6,8,10	4,8,12,16,20	4

Although the standard deviation of the underestimated final sigma is smaller than the overestimation, the absolute differences from the true sigma are greater.

GUARDRAILS

In Table 16, the critical values (or guardrails) for α = 0.05, 0.10, 0.20, and 0.40 are given for half-normal plot data when k = 31 and 63. These numbers would be used to test whether the largest contrast in any set of k's, is larger than the critical value for the given error rate, α. If so, it would be judged real.

If this information is to be applied, the investigator would first plot his data on a half-normal grid and remove all effects that obviously fall off the line formed by the estimated values of order statistics for k. Sensitive to the R spillover that is likely to occur, he may examine one or two ranks below that point to detect logical candidates for real effects. For example, if Effects A and AB were obviously large and Effect B was at a borderline rank, it would be tentatively considered real. Once the tentative and obvious real effects have been removed, the remaining k contrasts will be tested one

TABLE 16. GUARDRAILS FOR HALF-NORMAL PLOTS WHERE k = 31 and 63

(a)

k	Values of alpha .05	.10	.20	.40
31	3.326	3.056	2.759	2.408
30	3.305	3.041	2.740	2.407
29	3.328	3.047	2.739	2.400
28	3.317	3.037	2.735	2.376
27	3.300	3.015	2.715	2.369
26	3.345	3.033	2.723	2.359
25	3.261	2.973	2.668	2.335
24	3.243	2.967	2.661	2.313
23	3.280	2.968	2.645	2.296
22	3.257	2.952	2.619	2.269
21	3.225	2.934	2.615	2.255
20	3.251	2.945	2.605	2.234

(b)

k	Values of alpha .05	.10	.20	.40
63	3.446	3.211	2.960	2.659
62	3.483	3.236	2.967	2.662
61	3.473	3.209	2.951	2.655
60	3.490	3.234	2.971	2.660
59	3.473	3.217	2.949	2.648
58	3.458	3.202	2.944	2.636
57	3.431	3.186	2.919	2.622
56	3.445	3.212	2.947	2.634
55	3.467	3.209	2.933	2.626
54	3.439	3.194	2.918	2.611
53	3.410	3.171	2.907	2.605
52	3.441	3.178	2.906	[illegible]
51	3.427	3.173	2.894	[illegible]
50	3.398	3.158	2.891	[illegible]
49	3.415	3.157	2.883	[illegible]
48	3.422	3.166	[illegible]	[illegible]
47	3.405	3.143	[illegible]	[illegible]
46	3.427	3.173	[illegible]	[illegible]
45	3.387	[illegible]	[illegible]	[illegible]
44	3.386	[illegible]	[illegible]	

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PLOTS FOR INTERPRETI..(U) ESSEX CORP ORLANDO FL
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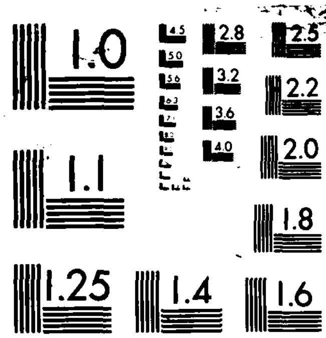

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at a time against the appropriate values in Table 16. For example, if six contrasts have already been removed from an original 63, then we would compare the next contrast with the critical values at rank 57 in Table 16. If it exceeds the appropriate value, then the next lower contrast is examined against the critical values at rank 56, and so forth.

The critical values were obtained using a Monte Carlo approach employing 10,000 runs. The raw contrasts are standardized using an estimate of σ, based on the slope of the smallest 0.85 of the contrasts which are tentatively assumed to be error contrasts. The number of contrasts from which the smallest 0.85 are taken is reduced by one each time the critical values at a new rank are calculated. This accounts for the fluctuation in values seen in the tables as k decreases.

At each rank, the critical values were calculated for the case where k is the highest rank for the k contrasts. Thus, each time, the test is made on only the largest one.

The guardrails are not provided for all ranks, only enough to test down to rank 44 when starting with $k = 63$, and down to rank 20 when starting with $k = 31$. This allows us to test for 20 real effects when initially there are 63 contrasts and 12 real effects when initially there are 31 contrasts. If more critical values are needed, they may be generated using the computer program for guardrails developed by Dr. David G. Weinman given in Appendix G. That program can be modified to generate guardrails for normal as well as half-normal plots.

It should not be overlooked that the use of guardrails at given alpha values is essentially a test to avoid Type 1 errors and the alpha indicates the probability that a nonreal effect will be included. But for screening purposes, it is the Type II error we wish to avoid. Therefore, in using these guardrails, one should probably use the larger alpha values, closer to the PER for the number of contrasts being examined; this increases the chances that a real effect will not be overlooked while allowing more null ones in. At the same time, the effective investigator will also inspect his data and rationally evaluate the effects tentatively considered to be real. This helps him determine which conditions will be included in the next block of the sequential data collection.

SECTION VII

RELEVANT PAPERS

A literature search discovered more than 100 reports in which Daniel's (1959) and Zahn's (1975a, b) work on half-normal plots were referenced (Appendix A). Many of these reports only describe how the half-normal plot has been put to use to interpret particular experimental data, generally without the use of sophisticated criteria to establish significance. A few, mostly from the statistical journals, examined the operating characteristics of these plots or supplied alternative approaches to some of the problems of interest in this report. While not intended to be a complete selection, 18 from that group were chosen for review on the basis of the following criteria:

1. It is relevant and appears to have some potential for improving the methodology of holistic human performance experiments.

2. It is not so overwhelmingly mathematical that its practical methodology is doubtful.

3. It illustrates alternative ways to use the normal or half-normal plot.

4. It proposes modified techniques in lieu of the normal or half-normal plot.

5. It provides illuminating nonmathematical discussions related to problems faced in this report.

6. It provides a list of references, some of which might prove fruitful to review further.

A short description of the selected references are reported below. It was beyond the scope of this project to study these in detail to determine if and how these techniques might be incorporated, along with the half-normal plot, into the methodology for a holistic approach to human performance research.

IDENTIFYING SIGNIFICANT EFFECTS IN UNREPLICATED EXPERIMENTS

Box, G. E. P., and Meyer, R. D. An analysis for unreplicated fractional factorials. Technometrics, 1986, 28, 11-18.

In the screening stage of industrial experimentation it is frequently true that the "Pareto Principle" applies. That principle states that a large proportion of process variation is associated with a small proportion of the process variables. In such circumstances of "factor scarcity," unreplicated fractional designs and other orthogonal arrays have frequently been effective when used as a screen for isolating preponderant factors. A useful graphical analysis due to Daniel (1959) employs normal rather than half-normal plotting for that purpose. A more formal analysis is presented in this paper, which may be used to supplement such plots and hence to facilitate the use of those unreplicated experimental arrangements.

The technique relies on reasonably accurate estimates of two parameters; the probability of an active effect, α, and the inflation factor of the standard deviation produced by the active effect, κ. The authors rely on some existing literature (mainly from the chemical industry) to obtain an approximate estimate of what these values might be. Employing a Bayesian approach, they compute the posterior probability that an effect is real. They feel that by utilizing modern numerical computing methods, the posterior probability calculations they propose can rapidly be made to provide visual displays of probability plots of the type they employed. From their analysis, they can determine whether reasonably reliable conclusions regarding whether effects are real are possible from the existing data or whether further experimentation is needed. They claim that the "conclusions drawn from [their] analysis are usually insensitive to moderate changes in α and κ, and [they] believe that little would be gained by attempting to be more precise" (p. 13). The extent to which this technique as an adjunct to Daniel's plotting procedure is applicable and practical in human performance research should be investigated further.

Box, G. E. P., and Meyer, R. D. Dispersion effects from the fractional designs. Technometrics, 1986, 28, 19-27.

The authors show how it is sometimes possible to use unreplicated fractional designs to identify factors that cause variability in performance as well as changes in average performance.

Holms, A. G., and Berrettoni, J. N. Chain-pooling ANOVA for two-level factorial replication-free experiments. Technometrics, 1969, 11, 725-746.

While Daniel concluded that the half-normal plot would be effective when only a few effects were significant, these investigations try to develop decision procedures to be used when more effects are significant and only a very small number of effects can be used to estimate the error variance. By testing the $2^f - r - 1$ mean squares in order of increasing magnitude, they use their procedure to try and estimate the number of null effects, which are then pooled into the denominator of the test statistic. The authors admit the technique is "good, even though it will not be shown analytically to be best." Appropriate risk functions are defined and several modifications of the suggested procedure are evaluated by Monte Carlo methods in terms of the risk functions.

OTHER APPLICATIONS FOR PLOTS

Barnett, V. The study of outliers: Purpose and model. Journal of the Royal Statistical Society, Series C (Applied Statistics), 1978, 27, 242-250.

The article discusses and illustrates the value of categorizing the different reasons outliers occur, the ways of handling outliers, and the models for outlier-generation. The question is raised as to whether outliers should be removed "as alien contaminants" or ignored until there is overt practical evidence that they are unrepresentative.

Hills, M. On looking at large correlation matrices. Biometrika, 1969, 56, 249-253.

Two graphical techniques are applied to a correlation matrix. The method of half-normal plotting is used to determine which coefficients are numerically too large to have come from zero population values. A z-transform is used before the absolute values of the correlated data are ordered and plotted. A visual clustering method is used also to select clusters of variable which have high positive correlations with each other.

Prew, R. D., Church, B. M. et al. Some factors limiting the growth and yield of winter wheat and their variation in two seasons. Journal of Agricultural Science, Cambridge, 1965, 104, 135-162.

The authors use the half-normal plot to examine the significance of 56 three-factor interactions, removed from the rest of the experimental data (p. 158-159). The square roots of the individual interaction mean squares were plotted.

Schweder, T., and Spjotvoll, E. Plots of P-values to evaluate many tests simultaneously. Biometrika, 1982, 69, 493-502.

By applying a normal-scores transform to both axes, a P-value is converted to a normal plot. The inverse transformation may also be carried out. The properties of the P-value plot are studied in some detail for such problems as: Comparing all pairs of means in a one-way layout; testing all correlation coefficients in a large correlation matrix; and evaluating all 2X2 subtables in a contingency table. Using Hills's (see above) data for comparison, they tended to prefer the P-value plot over the half-normal plot (p. 497-498) for three reasons: (1) The uniform probability transform (the P-value) is widely used when testing hypotheses; (2) It is slightly easier to evaluate the variance of a P-value than of a half-normal plot; (3) It is somewhat easier to fit a line to the P-value plot (because the tails of both axes are too stretched out in the half-normal plot).

Snee, R. D. Graphical analysis of process variation studies. Journal of Quality Technology, 1983, 15, 76-88.

Graphical techniques for analyzing the results of nested studies are presented and illustrated with examples. In addition to his "standard deviation control chart analysis," Snee also briefly covers the use of gamma probability plotting and data transformations (p. 85-88) to evaluate the homogeneity of a group of variances, that is, to determine whether they are all estimates of a single population variance. If the variances are homogeneous, then they will follow a χ^2 distribution which is a special case of the gamma family of distributions. The gamma plot operates like the probability plot. The half-normal distribution is a special case of the gamma distribution proposed by Wilk, Gnanadesikan, and Huyett (1962).

VARIATIONS ON THE NORMAL PLOT, GRAPHIC AND OTHERWISE

Andrews, D. F., and Tukey, J. W. Teletypewriter plots for data analysis can be fast: 6-line plots, including probability plots. Journal of the Royal Statistical Society, Series C (Applied Statistics), 1973, 22, 192.

Plots are generated which may be produced very quickly on a teletypewriter or similar remote terminal. The methods are useful for all displays which can be regarded as some form of the inspection of residuals. Versions for use in probability plotting, as an example not always thought of as an examination of residuals, are also given. One technique (mentioned in Daniel, 1959, p. 317) is described which avoids the need to fit a straight line: plot the log of the observed effect on an axis at 135° to the axis on which the log average quantiles are plotted (p. 199-200).

Box, G. E. P., and Tiao, G. C. A Bayesian approach to some outlier problems. Biometrika, 1968, 55, 119-129.

The contents of this paper may never be applicable to human performance research because the authors assume a precision and set of a priori assumptions that are not likely to be achieved. The paper is cited here, however, because: (1) A nonquantitative Bayesian-like approach underlies the holistic approach to human performance research; (2) This paper is a precursor to the Box and Meyer paper cited earlier; (3) Anything Box writes should be understood, even if it isn't immediately applicable.

Margolin, B. H. Systematic methods for analyzing $2^n\ 3^m$ factorial experiments with applications. Technometrics, 1967, 9, 245-259.

Two systematic procedures to facilitate the analysis of a complete $2^n\ 3^m$ factorial experiment are discussed. The methods are applicable when all quantitative three-level factors are equally spaced and when the contrasts involving qualitative three-level factors appear as if the three-level factors were in fact quantitative and equally spaced. Algorithm I systematizes the calculation of the factor effects for the $2^n\ 3^m$ series of designs. Algorithm II yields the set of fitted values, and hence the residuals, based on those factor effects which have been judged to be non-negligible. The two algorithms have additional and possibly more important uses in studying fractionated $2^n 3^m$ factorial experiments. Algorithm I can be used to facilitate the writing down of the cross-product matrix for any desired set of factor effects for a specified set of treatment comparisons. For the special case of the standard 2^{f-p} series of designs, the two algorithms can be used to find the set of defining contrasts corresponding to a given set of treatment combinations, or to find the set of treatment combinations corresponding to a given set of defining contrasts. That the analysis breaks the sources of the three-level factor and its interactions into single-degrees-of-freedom components suggests that the data might be inspected using the normal or half-normal plot (p. 249).

MacDonald, P. The analysis of a 2^n experiment by means of ranks. Journal of the Royal Statistical Society, Series C (Applied Statistics), 1971, 20, 259-270.

A method is proposed for the analysis of variance of ranked data from a factorial experiment of size 2^n which may be replicated or not. A test of the hypothesis that a given linear contrast of means is zero is based on the corresponding contrast of the ranks of the observations in each replicate. An appropriate critical region for the test is based on the rank sum test, which is equivalent to the new test under certain null hypotheses. Two other forms of the test are also suggested. Estimates of the population contrasts may be obtained by means of an ad hoc procedure for resolving the ambiguity in the means of the test statistic, together with normalization. The method is intended especially for the case where no quantitative observations can be made, or with the unreplicated experiment when quantitative measurements are possible. An example is given in which the half-normal plot is employed to identify the significant effects (p. 263-269).

GRAPHICAL TECHNIQUES IN STATISTICAL DATA ANALYSIS

Feder, P. I. Graphical techniques in statistical data analysis -- tools for extracting information from data. Technometrics, 1974, 16, 287-299.

A case study is presented that demonstrates the usefulness of graphical techniques in the analysis of data. Crossplotting, probability plotting, and graphical multiple comparison procedures are discussed. It is shown how graphical displays both motivate the use of and help interpret the results of more usual numerical techniques. The entire discussion is centered around the analysis of a particular set of experimental data involving a comparison of treatments. It is shown step by step how the various graphical techniques, used in conjunction with more classical techniques, extract the information contained in the data.

Nair, V. N. On the behavior of some estimators from probability plots. Journal of the American Statistical Association, 1984, 79, 823-831.

Fitting the straight line through a plot is used to estimate the standard error of the data. The author investigates the properties of those estimators. Estimators from weighted least square lines are considered and their asymptotic, finite-sample, robustness, and optimality properties are discussed. Included among these are the ordinary least squares estimators and estimators from least squares lines fitted after trimming or Winsorizing some of the extreme order statistics. The trimmed least squares estimators, with trimming properties reasonably chosen, provide a compromise between efficiency and robustness.

Wilk, M. B., and Gnanadesikan, R. Probability plotting methods for the analysis of data. Biometrika, 1968, 55, 1-17.

This paper describes and discusses graphical techniques, based on the primitive empirical cumulative distribution function (e.c.d.f.) and on quantile (Q-Q) plots, percent (P-P) plots, and hybrids of these, which are useful in assessing a one-dimensional sample, either from original data or resulting from analysis. Areas of application include: the comparison of samples; the comparison of distributions; the presentation of results on sensitivities of statistical methods; the analysis of results on sensitivities of statistical methods; the analysis of collections of contrasts and of collections of sample variances; the assessment of multivariate contrasts; and the structuring of analysis of variance mean squares. Many of the objectives and techniques are illustrated by example. Normal and half-normal plots, one class of six defined orthogonal analysis-of-variance situations, are discussed in Section 6.2, titled, "The univariate single degree of freedom case."

MULTIPLE COMPARISON PROCEDURES

Gnanadesikan, R., and Lee, E. T. Graphical techniques for internal comparisons amongst equal degree of freedom groupings in multiresponse experiments. Biometrika, 1970, 57, 229-237.

Probability plotting methods are given for two summary statistics derived from equal degree-of-freedom sum of product matrices. The methods are useful for graphical internal comparisons of the 'magnitudes' of the sum of products matrices. Possible applications with multiresponse data include the simultaneous assessment of all the main effects, or all of the interactions of the same order, in a factorial experiment with $m \geq 3$ levels for each factor, and the comparisons of several observed covariance matrices, for example, within-group covariance matrices in a multiresponse analysis of variance or discriminative analysis, each based on the same number of replicate observations. Illustrative applications are included.

Kurtz, T. E., Link, R. F., Tukey, J. W., and Wallace, D. L. Short-cut multiple comparisons for balanced single and double classifications: Part I, Results. Technometrics, 1965, 7, 95-161.

This paper includes a general and considered discussion of multiple comparison procedures: When they should and should not be used, the importance of confidence procedures and their advantages over significance procedures, choice among multiple comparisons confidence procedures, and choice and description of error rates. It does not attempt to compare multiple comparison and multiple decision procedures. The authors make an interesting distinction among experiments intended for "decision, significance, confidence, or selection."

Following the Kurtz paper in the same journal, comments on it were made by J. E. Jackson, p. 163-165, and F. J. Anscombe, p. 167-168; Kurtz et al. replied (p. 169) to Anscombe's comments.

Andrews, D. F., Gnanadesikan, R., and Warner, J. L. Transformation of multivariate data. Biometrics, 1971, 27, 825-840.

Methods which are extensions of Box and Cox's work (1964) are proposed for obtaining data-based transformations of multivariate observations to enhance the normality of their distribution and also possibly to simplify the model (e.g., improve additivity, homoscedasticity, etc.). Specifically, power transformations of the original variables are estimated to affect both marginal and joint normality. A method for improving directional normality is also described. Examples are included to illustrate some properties of the method with normal plots being employed to show the changes that occur in the data.

Miller, R. G., Jr. Developments in multiple comparisons, 1966-1976. Journal of the American Statistical Association, 1977, 72, 779-788.

A bibliography of 255 references is supplied including some references on graphical techniques. However, papers on ranking and selection were omitted as well as those on outlier detection.

SECTION VIII

CONCLUSIONS

From both aggregate and individual plots, whether quantified or interpreted by eye, whether normal or half-normal, our analyses clearly show that the use of plots cannot be relied upon as a certain and sole means of interpreting the results from a 2^f factorial-type experiment. They have their advantages but they also have their limitations, and cannot be considered the final word as a detection tool.

If we do not aspire to quantify our decisions regarding the probability that a particular contrast is real, the complications are less severe. We can look at the plots, and sometimes pick out most of the real effects. Sometimes, we won't be able to, but in all cases, it is the marginal real contrasts that will be the most difficult to detect and the marginal error contrasts that will be most difficult to reject.

The nature of the plots work against us. We have seen how the E- and the R-spillovers occur most frequently and over more ranks when the size of the real effects are smaller (and these are the marginal ones that need the most help in detecting), when the number of real effects increase (and for screening experiments we expect more rather than fewer real effects), and when the size of the experiment is smaller (which we prefer for the sake of economy).

It had been hoped that the guardrails would help reduce these problems. But this kind of quantification has proven to be little more than a numbers game, a quasi-scientific effort that still requires the investigator to use the same judgment he might have used had he not had the guardrails. For example, if we don't pick up enough real effects when a particular guardrail at one probability level is used, we are advised to raise the probability level until we're satisfied. After all, the guardrails set the error rate for calling an error contrast real, a decision which we are less concerned with in screening experiments than in calling the real contrast an error.

Calculating guardrails is a circular exercise. That means that to discover the correct answers, one must first know the correct answers. The guardrails that we need to help us decide which contrasts are real depend on an accurate estimate of the population sigma, inferred from one's data. Calculating the slope of the standardized error contrasts, or some part of them, is currently the best way to make this estimate. But to standardize the contrasts, one needs to know the population sigma, which is what we're trying to determine. As far as using the contrast closest to the 0.683 probability level for the half-normal plot (as originally suggested by Daniel) or 0.34 and 0.84 probability levels for the normal plot as the initial estimate of sigma, that can only lead to incorrect estimates unless there are a very few real effects, a condition not to be expected in screening experiments. We have seen how much a wrong guess distorts the final sigma estimate.

But the problem of circularity doesn't stop there. The slope that will serve as the best estimate of the population sigma is the plot relating the standardized error contrasts to the estimated values of normal order statistics for the correct number of error contrasts. But we can only know the correct number of error contrasts if we know the correct number of real contrasts, the purpose of this entire exercise in the first place.

Guardrails just don't add anything that might not be achieved using an informed rational approach, some appreciation of the E- and R-spillover for the particular experimental space, and a liberal attitude toward the inclusion of some error terms. A cursory examination of the literature in which plots are employed seems to suggest that today most investigators agree with this point; at least those who use probability plots don't use guardrails. The ineffectiveness of guardrails does not eliminate the usefulness of probability plots. F-tests have many of the same weaknesses in practice as plots have and have not some of their advantages. The author still believes that when used judiciously plots are still the best way to interpret the results from a screening experiment.

What advantages do probability plots have for identifying the critical factors? For one thing, they require an organization of the data in a way that may facilitate interpretation. By ordering the contrasts, one can frequently infer from the larger ones which marginal ones are also likely to be real. If the proper preliminary analysis is done before the experiment begins, a rational approach is unlikely to be any worse than using guardrails to make detection decisions and may be a whole lot better. For another thing, plots are a quick way to examine one's data. One doesn't need a computer, only a table of normal order statistics. Then too, when one is working with unreplicated designs, the plots can be used with no major assumptions, whereas if one still insists on using an analysis of variance for significance testing, it would be necessary to dream up an imaginary error variance, probably based on some combination of higher-order interaction effects, which aren't likely to be available in the early stages of a screening experiment anyway. As we move through the sequential stages of an experiment, adding blocks to unravel confounded effects, the effectiveness of plots improve because of the larger n. As an added advantage, probability plotting can be another way to examine one's data for abnormalities.

But should we use normal or half-normal plots? There were some indications that the normal plot had advantages in terms of the R-spillover over the half-normal, but not much. On the other hand, the half-normal eliminates the need to check both ends of the continuum since the absolute effects are all positive. Of course, once data are obtained, they can be plotted in both ways. Inspection of both plots can't hurt and may provide clues to borderline cases. It was seen that if one combined the data from a normal and a half-normal plot when making an initial estimate of sigma for purposes of standardizing the data, the average of the final estimate of sigma from both gave a more accurate estimate than either alone because of the reciprocal relationship.

From our literature search, we noted that investigators continue to use the plot as an aid to interpreting different kinds of data and for different purposes. Some continue to propose alternative solutions to problems for which probability plots were originally intended to solve.

ADDITIONAL EFFORTS

Some problems that might be of interest to pursue (although to do so may not necessarily be cost-effective) would be to examine the effect of confounding on the effectiveness of probability plots. With screening experiments, main effects are confounded with all odd higher-order interaction effects and two-factor interaction effects are confounded not only with two-factor interactions, in some cases, but also the even higher-order interaction effects. The plot only treats the combination as a single degree of freedom. But it is not clear whether or how the probability of detecting critical factors is affected when four effects are confounded versus eight effects in an aliased interaction string of a fractional factorial.

Another thing that might be done would be to expand the relatively simple relationship discovered between r and d and the R-spillover and include n and to transform the data so as to improve the linearization of the equation. [Note: This is being done.] The relation between R-spillover (and E-spillover) to contrast degradation still lends itself to quantification provided one can find pragmatic advantages in knowing how to use this information to interpret the individual experiment. Also, finding ways to relate results when the sizes of multiple real effects are different rather than equal, as in most of our examples, might make tables of R-spillover have greater utility.

Finally, it would be useful to find what has and hasn't been done regarding the use of probability plots to (1) examine the quality of one's data, and (2) interpret the results from multifactor-multivariate experiments.

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APPENDIX A

LITERATURE SEARCH

A search was made in the Science Citation Indexes from 1955 to 1986 and in the CompuMath Citation Indexes from 1981 to 1986 for any references to Daniel's (1959) and Zahn's (1975a,b) papers on half-normal plots.

The list below provides the authors, journals, and location within the journals of articles which referred to Daniel's and Zahn's papers. None of the report titles are given, although they may be found, if desired, in the Sources Indexes which accompany the Science Citation and CompuMath Citation Indexes. Even in its abbreviated form, however, this list provides enough information for a user to go directly to the appropriate journals to find an article.

The list is useful for those who wish to see how normal and half-normal plots have been used. A majority of the articles listed below describe applications rather than analyze or otherwise discuss or examine the operating characteristics of these plots. It cannot be assumed that this list is complete; for example, it does not include any references from government or industrial publications. The list does indicate the impact Daniel's paper has had on the interpretation of experimental data. That fact is particularly interesting in the light of Zahn's criticisms and the fact that Zahn's paper -- undoubtedly more technically correct than Daniel's -- is only referenced in these same volumes a few times. All of the listed papers were found in the Science Citation Index except those marked with an #, which came from the CompuMath Citation Index.

The references are organized according to the years contained in the individual volumes of these indexes.

ARTICLES REFERENCING DANIEL'S (1959) PAPER ON HALF-NORMAL PLOTS

Index Vol. Year(s)	1st Author	Journal	Volume	1st Page	Year
1955-1969					
	Cox, DR	Technomet	9	481	67
	Draper, NR	Can Math B	11	475	68
	Fedorov, VD	Dan SSSR	188	913	69
	Fienberg, SE	Appl Stat	18	153	69
	Gorman, JW	Technomet	8	27	66
	Govindar, VM	Ind Eng PDD	7	573	68
	Healy, MJR	Appl Stat	17	157	68
	Hills, M	Biometrika	56	249	69
	Holms, AG	Technomet	11	725	69
	Kurtz, TE	Technomet	7	95	65
	Margolin, BH	Technomet	9	245	67
	Myers, MH	J Chron Dis	19	923	66
	Shapiro, SS	Biometrika	52	591	65
	Stowe, RA	Ind Eng Ch	58	36	66
	Stowe, RA	Ind Eng Ch	61	11	69
	Webb, SR	Technomet	10	535	68
	Wilk, MB	Biometrika	55	1	68
1955-1964					
	Addelman, S	Technomet	4	21	62
	Addelman, S	Technomet	6	365	64
	Banerlee, KS	Ann Math St	34	1068	63
	Box, GEP	Technomet	4	301	62
	Draper, NR	Biometrics	20	443	64
	Elandt, RC	Technomet	3	551	61
	Hunter, JS	Technomet	6	41	64
	Leone, FC	Technomet	3	543	61
	Stevens, CD	J Pharm Exp	141	267	63
	Sweeny, R	Ind Eng Ch	53	329	61
	Truax, HM	J Am Oil Ch	37	650	60
	Tukey, JW	Ann Math St	33	812	62
	Wilk, MB	Ann Math St	36	613	64
	Wilk, MB	J Am Stat A	58	152	63
	Wilk, MB	Technomet	4	1	67
1970-1974					
	Abdulrah, YA	Chem Eng Sc	28	1273	73
	Andrews, DF	Biometrics	27	825	71
	Andrews, DF	J Roy Sta C	22	192	73
	Anscombe, FJ	Am Statistn	27	17	73
	Congdon, CC	J Nat Canc	45	1055	70
	Dyer, DD	Technomet	15	489	73
	Egorov, NS	Mikrobiolog	42	863	73
	Evans, DA	J Roy Sta A	136	153	73
	Feder, PI	Technomet	16	287	74
	Fleming, AF	Med J Aust	2	429	74
	Gnanades, R	Ann Math St	41	292	70
	Gnanades, R	Biometrika	57	229	70

Index Vol. Year(s)	1st Author	Journal	Volume	1st Page	Year
1970-1974 (cont'd)					
	Gnanades, R	J Roy Sta B	32	88	70
	Goldsmith, PL	J Roy Sta C	22	141	73
	Goswami, BC	Text Res J	42	605	72
	Guttman, I	Technomet	15	723	73
	MacDonal, P	J Roy Sta C	20	259	71
	Mallows, CL	Ann Math St	43	508	72
	Miller, B	Tappi	57	102	74
	Mueller, FX	J Paint Tech	43	54	71
	Munford, AJ	J Roy Sta C N	21	351	72
	Rubin, IB	Analyt Chem	43	717	71
	Scardino, FL	Text Res J	40	932	70
	Sparks, DN	Appl Stat N	19	192	70
	Winchest, SC	Text Res J	40	458	70
	Zahn, DA	Biometrics M	27	773	71
1975-1979					
	Barnett, V	J Roy Sta A	139	318	78
	Barnett, V	J Roy Sta C	27	242	78
	Beck, T	Act Techn M	81	313	75
	Bennett, DR	J Anim Sci	45	768	77
	Coldwell, RL	IEEE Ind Ap	14	175	78
	Egorov, MS	Microbiolog	44	206	75
	Egorov, MS	Microbiolog	45	87	76
	Fienberg, SE	Am Statistn	33	165	79
	Gentleman, JF	Technomet	17	1	75
	Gogoleva, EV	Microbiolog	45	690	76
	Gruber, CM	J Med	10	65	79
	Gupta, SP	Soc Pet E J	19	166	79
	Muck, PM	J Water PC	49	2411	77
	Kale, BK	Sankmya B	38	356	76
	Lamb, GER	Text Res J	45	452	75
	Mead, R	Biometrics R	31	803	75
	Milko, ES	Microbiolog	46	395	77
	Murphy, TO	Chem Eng	84	168	77
	Rutella, GS	Clin Chem	25	1954	79
	Rosner, B	Technomet	19	307	77
	Rubin, IB	Analyt Chem R	51	541	79
	Sachs, L	Klin Woch R	55	973	77
	Shimalla, CJ	Text Res J	46	313	76
	Stavig, GR	Psychol B	83	236	76
	Turnbull, BW	Biometrics	34	555	78
	Wilkens, WD	IEEE El Ins	12	60	77
	Zahn, DA	Technomet	17	189	75
	Zahn, DA	Technomet	17	201	75

Index Vol. Year(s)	1st Author	Journal	Volume	1st Page	Year
1980					
	Gusev, NV	Microbiolog	49	19	80
	Holms, AG	Comm Stat B	9	51	80
	Lam, CL	J Food Sci	45	1720	80
	Maksimov, VN	Microbiolog	49	186	80
1981					
	Archer, RH	Eur J Appl	12	46	81
	Darby, SC	J Roy Sta A	144	296	81
	Eldin, SH	J Appl Poly M	26	1431	81
	Jurgenson, IA	Parazitolog	15	38	81
1982					
	Bradu, D	Technomet	24	103	82
	Schweder, T	Biometrika	69	493	82
	# Stirling, WD	Statistica	31	211	82
	Wilkinson, SA	J Food Sci	47	844	82
1983					
	Cardone, MJ	J Aoac	66	1257	83
	Carroll, MB	Am Statistn E	37	31	83
	Cochrane, RL	Biol Reprod	28	134	83
	Huck, PM	Water Res	17	1403	83
	Malajczu, N	Ann Ap Biol	103	57	83
	Snee, RD	J Qual Tech	15	76	83
1984					
	Cox, DR	Int Stat R R	52	1	84
	Kotze, TJV	Appl Stat	33	215	84
	# Mauro, CA	Manag Sci	30	209	84
	Nair, VN	J Am Stat A	79	823	84
	Ziegel, ER	Technomet D	26	98	84
1985					
	Funk, W	Thin Sol Fi	128	45	85
	Kaitala, S	Ecol Bull		125	84
	Prew, RD	J Agr Sci	104	135	85
1986					
	Box, GEP	Technomet	28	11	86

ARTICLES REFERENCING ZAHN'S (1975a,b) PAPERS ON HALF-NORMAL PLOTS

Index Vol. Year(s)	1st Author	Journal	Volume	1st Page	Year
1975-1979					
Miller, RG	J Am Stat A	72	779	077	77
Sachs, L	Klin Woch	55	973	077	77
1980					
Dixon, WJ	Ann R Pharm	20	441	080	80
1981-1986					
None listed					

APPENDIX B

EXPECTED VALUES OF NORMAL AND HALF-NORMAL ORDER STATISTICS FOR k = 31, 63, and 127

NORMAL ORDER STATISTICS

#	ORDER STATISTIC
1	-2.05647
2	-1.63167
3	-1.38269
4	-1.19804
5	-1.04709
6	-0.91688
7	-0.80066
8	-0.69438
9	-0.59546
10	-0.50206
11	-0.41287
12	-0.32686
13	-0.24322
14	-0.16126
15	-0.08037
16	0.00000
17	0.08037
18	0.16126
19	0.24322
20	0.32686
21	0.41287
22	0.50206
23	0.59545
24	0.69438
25	0.80065
26	0.91689
27	1.04709
28	1.19804
29	1.38269
30	1.63167
31	2.05647

#	ORDER STATISTIC
1	-2.33781
2	-1.95626
3	-1.73905
4	-1.58179
5	-1.45605
6	-1.34981
7	-1.25699
8	-1.17388
9	-1.09820
10	-1.02834
11	-0.96317
12	-0.90188
13	-0.84380
14	-0.78844
15	-0.73539
16	-0.68436
17	-0.63504
18	-0.58724
19	-0.54073
20	-0.49537
21	-0.45101
22	-0.407[illegible]3
23	-0.36480
24	-0.32273
25	-0.28122
26	-0.24019
27	-0.19957
28	-0.15927
29	-0.11923
30	-0.07938
31	-0.03966
32	0.00000
33	0.03966
34	0.07938
35	0.11923
36	0.15927
37	0.19957
38	0.24019
39	0.28122
40	0.32273
41	0.36480
42	0.40753
43	0.45101
44	0.49536
45	0.54073
46	0.58724
47	0.63504
48	0.68436
49	0.73539
50	0.78844
51	0.84380
52	0.90188
53	0.96317
54	1.02833
55	1.09820
56	1.17388
57	1.25698
58	1.34981
59	1.45605
60	1.58178
61	1.73905
62	1.95626
63	2.33781

#	ORDER STATISTIC
1	-2.59185
2	-2.24250
3	-2.04751
4	-1.90849
5	-1.79870
6	-1.70701
7	-1.62779
8	-1.55772
9	-1.49450
10	-1.43669
11	-1.38338
12	-1.33376
13	-1.28721
14	-1.24331
15	-1.20171
16	-1.16204
17	-1.12415
18	-1.08776
19	-1.05286
20	-1.01913
21	-0.98650
22	-0.95496
23	-0.92428
24	-0.89449
25	-0.86544
26	-0.83714
27	-0.80947
28	-0.78239
29	-0.75589
30	-0.72996
31	-0.70447
32	-0.67941
33	-0.65478
34	-0.63058
35	-0.60669
36	-0.58317
37	-0.55997
38	-0.53704
39	-0.51443
40	-0.49207
41	-0.46995
42	-0.44805
43	-0.42635
44	-0.40488
45	-0.38359
46	-0.36245
47	-0.34149
48	-0.32068
49	-0.30000
50	-0.27947
51	-0.25902
52	-0.23869
53	-0.21847
54	-0.19833
55	-0.17828
56	-0.15829
57	-0.13837
58	-0.11850
59	-0.09868
60	-0.07889
61	-0.05914
62	-0.03942
63	-0.01971
64	0.00000
65	0.01971
66	0.03942
67	0.05914
68	0.07889
69	0.09868
70	0.11850
71	0.13837
72	0.15829
73	0.17828
74	0.19833
75	0.21847
76	0.23870
77	0.25903
78	0.27947
79	0.30000
80	0.32068
81	0.34150
82	0.36245
83	0.38360
84	0.40489
85	0.42636
86	0.44805
87	0.46994
88	0.49208
89	0.51443
90	0.53704
91	0.55997
92	0.58317
93	0.60670
94	0.63058
95	0.65479
96	0.67941
97	0.70447
98	0.72997
99	0.75589
100	0.78239
101	0.80948
102	0.83714
103	0.86544
104	0.89451
105	0.92427
106	0.95497
107	0.98651
108	1.01915
109	1.05286
110	1.08776
111	1.12413
112	1.16205
113	1.20171
114	1.24333
115	1.28722
116	1.33375
117	1.38340
118	1.43670
119	1.49449
120	1.55775
121	1.62778
122	1.70698
123	1.79873
124	1.90851
125	2.04752
126	2.24250
127	2.59185

APPENDIX B (cont'd)

HALF-NORMAL ORDER STATISTICS

Number	Order Statistic
1	0.037755
2	0.077344
3	0.116146
4	0.155737
5	0.196288
6	0.236848
7	0.277176
8	0.318803
9	0.361311
10	0.402889
11	0.446393
12	0.490765
13	0.535489
14	0.580973
15	0.629700
16	0.679548
17	0.732111
18	0.786062
19	0.842622
20	0.901025
21	0.961400
22	1.026500
23	1.096640
24	1.173480
25	1.256180
26	1.348710
27	1.457650
28	1.584930
29	1.744760
30	1.959020
31	2.342680

Number	Order Statistic
1	0.019230
2	0.039096
3	0.058495
4	0.077851
5	0.097564
6	0.117018
7	0.137275
8	0.156933
9	0.176493
10	0.196258
11	0.216200
12	0.235971
13	0.256196
14	0.276680
15	0.297813
16	0.318330
17	0.339698
18	0.360498
19	0.381368
20	0.402650
21	0.423443
22	0.445411
23	0.467422
24	0.489720
25	0.512424
26	0.534829
27	0.557291
28	0.579992
29	0.603416
30	0.627738
31	0.651868
32	0.676398
33	0.701125
34	0.727618
35	0.753351
36	0.779807
37	0.807111
38	0.834626
39	0.862614
40	0.891383
41	0.921166
42	0.952432
43	0.983861
44	1.016860
45	1.050560
46	1.085090
47	1.121530
48	1.159500
49	1.199540
50	1.241350
51	1.285290
52	1.332120
53	1.383010
54	1.435900
55	1.493260
56	1.555480
57	1.624930
58	1.704560
59	1.797090
60	1.908770
61	2.047880
62	2.240550
63	2.592380

APPENDIX B (cont'd)

HALF-NORMAL ORDER STATISTICS

Number	Order Statistic
1	0.009809
2	0.019836
3	0.029776
4	0.039751
5	0.049482
6	0.059341
7	0.069374
8	0.079189
9	0.089033
10	0.098757
11	0.108474
12	0.118526
13	0.128607
14	0.138253
15	0.148170
16	0.158008
17	0.167908
18	0.177665
19	0.187725
20	0.197852
21	0.207862
22	0.217746
23	0.227816
24	0.237906
25	0.247963
26	0.257876
27	0.268187
28	0.278347
29	0.288580
30	0.298931
31	0.309051
32	0.319467
33	0.329850
34	0.340470
35	0.350732
36	0.361337
37	0.371741
38	0.382275
39	0.392913
40	0.403483
41	0.414017
42	0.424991
43	0.436135
44	0.446763
45	0.457691
46	0.468713
47	0.479703
48	0.490531
49	0.501775
50	0.512986
51	0.524642
52	0.535977
53	0.547388
54	0.558796
55	0.570297
56	0.581697
57	0.593361
58	0.604956
59	0.616701
60	0.628569
61	0.640321
62	0.652393
63	0.664481
64	0.676715
65	0.689347
66	0.701778
67	0.714480
68	0.727106
69	0.740169
70	0.753378
71	0.766354
72	0.779366
73	0.792861
74	0.806778
75	0.820474
76	0.834340
77	0.848549
78	0.862394
79	0.876830
80	0.891408
81	0.905992
82	0.920773
83	0.935736
84	0.951191
85	0.967161
86	0.983027
87	0.998870
88	1.014870
89	1.031260
90	1.048090
91	1.065140
92	1.083030
93	1.100680
94	1.119210
95	1.137670
96	1.156790
97	1.176400
98	1.195660
99	1.215710
100	1.236340
101	1.257680
102	1.279600
103	1.302500
104	1.325520
105	1.349420
106	1.373820
107	1.399130
108	1.425780
109	1.453400
110	1.482160
111	1.512110
112	1.543380
113	1.576830
114	1.612090
115	1.649600
116	1.689460
117	1.731980
118	1.776400
119	1.826620
120	1.881770
121	1.943290
122	2.012240
123	2.094940
124	2.193610
125	2.318260
126	2.497170
127	2.828630

APPENDIX C

PROGRAM FOR GENERATING DATA BASES AND FOR PERFORMING SUBSEQUENT ANALYSES USED IN THIS REPORT*

This program constructs experimental designs of the 2^f (or 2^{f-p}) type and will produce summary or "estimated" expected results for any combination of real and nonreal effects with minor modifications by the user. The program also produces a variety of other statistics and output which can be used to investigate the operating characteristics of multifactor experiments and normal plots. The program works by first constructing an experimental design of the 2^f type with the matrix filled with -1s and +1s to represent the (2) levels of the factors. Then random normal values (N(0,1)) are generated and "assigned" as data points to the experiment. There are options for assigning more than one data point to each cell of the design matrix.

The effects or contrasts for the experiment are then computed. Real or "true" effect values are then added to the estimated nonreal contrasts and the effects are then sorted according to size. This process is repeated some number of times (until desired accuracy is achieved -- 5000 in this report), and summary statistics giving means and standard deviations for the sorted contrasts are computed. The program also has subroutines for computing standard normal order statistics for any number of ordered values and these can be printed for both real and nonreal effects for comparison purposes. In its current form, certain selected summary statistics are printed to a disc file which can then be subjected to further analysis using other statistical packages. In this report, the normal plot routines in the BMDP statistical package were used to plot the obtained ordered values. This method was used to obtain normal plots for the outcome from a single experiment, the ordered summary statistics for all results, and the ordered summary statistics for intermediate results. The user can modify this output as necessary to conduct supplemental analyses. Currently, the program will also calculate slope statistics for various portions of the normal plot.

---

* Questions regarding this program should be addressed to Dr. Daniel P. Westra, Essex Corporation, 1040 Woodcock Road, Orlando, FL 32813.

The user must input the following to the program:

1. The size of the experimental design (expressed as 2^f).
2. The number of real effects.
3. The magnitude of each real effect.
4. Number of data points per cell (default to one).

```
C
C
C
C
C     THIS PROGRAM WAS WRITTEN ON A VAX-780 USING
C     VAX FORTRAN-77+.  THE FILE NAME WES.FOR .
C
C     TO COMPILE:  FOR WES
C
C     TO TASK BUILD: LINK WES
C
C
C     INPUTS: THE INPUTS TO THIS PROGRAM ARE
C
C                 1) THE SIZE OF THE DESIGN MATRIX.
C             2) THE NUMBER OF REAL EFFECTS.
C             3) THE MAGNITUDE OF EACH REAL EFFECT.
C
C     OUTPUT: THERE ARE FIVE FILES THAT ARE CREATED
C
C             1) SIMON.DAT - THIS FILE HAS SUMMARY DATA AFTER
C                            EACH 100 RUNS.
C
C             2) WES.DAT - THIS FILE IS THE FINAL SUMMARY AFTER
C                          5000 RUNS.
C
C             3) SLOPE.DAT - THIS FILE WAS CREATED TO BE AN
C                            INPUT FILE TO THE BMDP PACKAGE
C                            5D TO CALCULATE THE PLOT OF THREE
C                            SIGMA METHODS.
C
C             4) SIMONBMD.DAT - THIS FILE WAS CREATED TO BE AN INPUT
C                               FILE TO THE BMDP PACKAGE 5D TO PLOT
C                               NEWE AFTER EACH 100TH RUN.
C
C             5) ZMD.DAT - THIS FILE WAS CREATED TO BE AN INPUT
C                          FILE TO THE BMDP PACKAGE 5D TO PLOT NEWE
C                          AFTER 5000 RUNS.
C
C
C
C
C
C
C
       IMPLICIT REAL*4 (M)
C
              DIMENSION D(128),E(127),P(128,127),EN(127),
     X        NS(128),EN1(127),E1(127),YEN(127),YEN2(127)
       DIMENSION ESS(127),RN(128),MAG(32),YENA(127),YENA2(127)
C
       INTEGER P,COUNT(127),T30,T50,T70,F30,L30,F50,L50,F70,L70,
     X           EFF(127),POSEFF,NEGEFF,K1,K2,NEWK,P30E,J30E,P50E,J50E
```

```
C
      REAL*4 NEWE(127),D30,D50,D70,NEWD,COL1(127),COL2(127)
      REAL*4 ONE(30),TWO(29),THREE(28),FOUR(27),FIVE(26)
      REAL*4 SIX(25),SEVEN(24),NEWYEN(127),NEWYEN2(127)
      REAL*4 ZMDZN(127),ZMDZE(127),ZMDZE1(127),ZEIGHT(127)
      REAL*4 ZEIGHT1(127),MAT(31,50)
C
      LOGICAL TEST(127)
C
C     SIGN MATRIX USED AS BASIS FOR GENERATING OTHER SIZE FACTORIALS.
C
      DATA ((P(I,J),J=1,3),I=1,4) /-1.,-1.,1.,1.,-1.,-1.,-1.,1.,
     X      -1.,1.,1.,1./
C
      OPEN(UNIT=11,TYPE='NEW',NAME='SLOPE.DAT.')
C
      NR=5000
      SEED=SECNDS(0.0)
C
C     NMAX MUST BE INPUT BY USER . IT SPECIFIES SIZE OF DESIGN
C     ACCORDING TO 2**NMAX.
C     NSCELL ALSO USER INPUT. GIVES NO. SUBS PER CELL.
C
      WRITE(5,2)
2     FORMAT(' ENTER SIZE OF DESIGN MATRIX ')
      ACCEPT 3,NMAX
3     FORMAT(I2)
      NCMAX=NMAX-1
      NFCON=2**NMAX
      NFEFF=NFCON-1
      NSCELL=1
C
      IF (NMAX .EQ. 5) THEN
          F30 = 12
       L30 = 20
       D30 = 9.
       F50 = 9
       L50 = 23
       D50 = 15.
       F70 = 6
       L70 = 26
       D70 = 21.
       NEWD = 0.3535533
      ELSE IF (NMAX .EQ. 6) THEN
       F30 = 23
       L30 = 41
       D30 = 19.
       F50 = 17
       L50 = 47
       D50 = 31.
       F70 = 10
       L70 = 54
       D70 = 45.

       NEWD = 0.25
       END IF
C
       NTEMP=NFCON*NSCELL
C
C      GENERATE THE DESIGN MATRIX
C
       DO 101 N1=2,NCMAX
         NC1=2**N1
         NC2=NC1*2
         NST=NC1+1
         NE1=NC1-1
         NE2=NC2-1
C
C      DUPLICATE EXISTING MATRIX
C
         DO 11 I=NST,NC2
           DO 11 J=1,NE1
             P(I,J)=P(I-NC1,J)
11           CONTINUE
C
C      GENERATE NEXT FACTOR
C
         DO 15 I=1,NC1
           P(I,NC1)=-1.
15           CONTINUE
         DO 16 I=NST,NC2
           P(I,NC1)=1.
16           CONTINUE
C
C      MULT TERMS BY NEW FACTOR
C
         DO 31 I=1,NC2
           DO 31 J=NST,NE2
             P(I,J)=P(I,NC1)*P(I,J-NC1)
31           CONTINUE
101    CONTINUE
C
```

```
      DO 20 I=1,NTEMP
        NS(I)=I
20        CONTINUE
      DO 21 J=1,NFEFF
              ESS(J)=0.
         E1(J) = 0.
         COL1(J)=0.
         ZMDZE(J)=0.
         ZMDZE1(J)=0.
         ZMDZN(J)=0.
         COL2(J)=0.
         E(J)=0.
21        CONTINUE
      SLOPE30 = 0.
      SLOPE50 = 0.

      SLOPE70 = 0.
      T30 = 0
      T50 = 0
      T70 = 0
      K1 = 0
      K2 = 0
C
      DO 91 I=1,32
          MAG(I) = 0.0
91    CONTINUE
      WRITE(5,4)
4     FORMAT(' ENTER NUMBER OF REAL EFFECTS ')
      ACCEPT 3,F
      DO I = 1,F
        TYPE*,I
        WRITE(5,5)
5       FORMAT(' ENTER MAGNITUDE OF EFFECTS ')
        ACCEPT 6,MAG(I)
6       FORMAT(F7.4)
        IF(MAG(I) .GT. 0.)POSEFF=POSEFF+1
        IF(MAG(I) .LT. 0.)NEGEFF=NEGEFF+1
      END DO
C
C     THE FOLLOWING IF BLOCK IS USED TO ASSIGN VARIABLES
C     THE CORRECT VALUE DEPENDING OF THE THE DESIGN SIZE
C     AND IF A 30 OR 50 PERCENT SLOPE ABOUT THE CENTER
C     IS NEEDED.
C
      IF (NMAX .EQ. 5) THEN
        IF (F .LE. 10) THEN
          P30E = 8
          J30E = 16
          P50E = 5
          J50E = 19
        ELSE IF (F .EQ. 12) THEN
          P30E = 6
          J30E = 14
          P50E = 3
          J50E = 17
        ELSE IF (F .EQ. 16) THEN
          P30E = 4
          J30E = 12
          P50E = 1
          J50E = 15
        END IF
C
      ELSE IF (NMAX .EQ.6) THEN
        IF (F .EQ. 8) THEN
          P30E = 19
          J30E = 37
          P50E = 13
          J50E = 43
        ELSE IF (F .EQ. 12) THEN

          P30E = 17
          J30E = 35
          P50E = 11
          J50E = 41
        ELSE IF (F .EQ.16) THEN
          P30E = 15
          J30E = 33
          P50E = 9
          J50E = 39
        END IF
      END IF
```

```
C
C     THIS WILL ECHO CHECK THE POSITIVE AND NEGATIVE
C     VALUES OF THE REAL EFFECTS.
C
      TYPE*,POSEFF
      TYPE*,NEGEFF
C
      K1 = NFEFF-POSEFF
      K2 = NFEFF-F
C
C     ORDER STAT. FOR REAL AND NON-REAL EFFECTS
C
      DO I = 1,NFEFF
        CALL SCOR(I,NFEFF,SN,VAR)
        YEN(I) = SN
        YEN2(I)= VAR**.5
        NEWYEN(I) = SN            !ORDER STAT. FOR NFEFF
        NEWYEN2(I) = VAR**.5      !STD.DEV. FOR ORDER STAT.
      END DO
C
C     ORDER STAT. FOR NON-REAL EFFECTS
C
      DO I=1,K2
        CALL SCOR(I,K2,SN,VAR)
        YENA(I) = SN
        YENA2(I) = VAR**.5
      END DO
C
C     ORDER STAT. FOR REAL EFFECTS
C
      DO I=1,F
        CALL SCOR(I,F,SN,VAR)
        ZEIGHT(I) = SN
        ZEIGHT1(I)= VAR**.5
      END DO
C
      CALL SORT(YEN,NFEFF)
      CALL SORT(YENA,K2)
      CALL SORT(NEWYEN,NFEFF)
      CALL SORT(ZEIGHT,F)
C
      IF(NEGEFF .EQ. 0)GOTO 9


 C
      DO I=1,NEGEFF                    !LOOP ONE
        EFF(I) = F
        YEN(I) = ZEIGHT(I) + (MAG(I)/NEWD)
        YEN2(I)= ZEIGHT1(I)
      END DO
 C
      IF (NEGEFF .GT. 0)NEGEFF=NEGEFF+1
 C
  9   IF (NEGEFF .EQ. 0)NEGEFF=1
 C
      DO I=NEGEFF,K1                   !LOOP TWO
       EFF(I) = K2
       YEN(I) = YENA(I-(NEGEFF-1))
       YEN2(I)= YENA2(I-(NEGEFF-1))
      END DO
 C
      K1 = K1 + 1
      DO I=K1,NFEFF
        EFF(I) = F
        YEN(I) = ZEIGHT(I-K2) + (MAG(I-K2)/NEWD)
        YEN2(I)= ZEIGHT1(I-K2)
      END DO
 C
 C
 C    START OF LOOP FOR THE 10K(OR WHATEVER) RUNS
 C
 C
      ITEMP=100
      NEWK=1
 C
      DO 100 KK=1,NR
 C
 C    RANDOM DRAW FROM N(0,1)
 C
        CALL RNORM01(NTEMP,RN,SEED)
        DO 509 I=1,NTEMP
          D(I)=RN(I)
509       CONTINUE
C
631     DO 25 I=1,NFEFF
          TEST(I) = .FALSE.
          EN1(I) = 0.
          EN(I)=0.
25        CONTINUE
```

```
C
C     RANDOM ASSIGNMENT OF "SUBJECTS" TO CONDITIONS
C
        NTEM2=NTEMP-1
        DO 30 I=1,NTEM2
          IP=NTEMP-I+1
          POS=IP
          RAN = MTH$RANDOM(SEED)


         ICM=RAN*POS+1
         IS=NS(IP)
         NS(IP)=NS(ICM)
         NS(ICM)=IS
30         CONTINUE
C
C    NUMBERS REPRESENTING SUBJECTS MEAN LEVEL INPUT AND TALLIED
C
       DO 40 J=1,NFEFF
         K=1
         DO 40 I=1,NFCON
           C=0.
C
C    PROVISION TO PUT IN MORE THAN 1 SUB PER CELL.
C
           K2=K+NSCELL-1
           DO 43 K1=K,K2
             II=NS(K1)
             C=C+D(II)
43             CONTINUE
           C=C/NSCELL
           K=K2+1
           EN(J)=EN(J)+P(I,J)*C
40         CONTINUE
C
       JJ2=NFCON/2
C
C     USE ABS VALUES OF MEAN DIFFERENCES HERE IF DESIRED.
C
       DO 41 I=1,NFEFF
         EN(I)=(EN(I))/FLOAT(JJ2)
41         CONTINUE
C
       DO K = 1,F
         EN(K) = EN(K) + MAG(K)
       END DO
       DO I = 1,NFEFF
         EN1(I) = EN(I)
       END DO
C
C    SORT EFFECT SIZES
C
       CALL SORT(EN,NFEFF)
C
C
C
C    NOW COMPUTE SLOPE BASED ON CENTER POINTS.  WE WILL USE
C    29% , 55% , AND 68% OF THE DATA ABOUT THE MIDPOINT
C    OF THE SORTED ARRAY.
C
C    FIRST: 29%
C
       I30 = 0


   SUM30XY = 0.
   SUM30X = 0.
   SUM30XX = 0.
   SUM30Y = 0.
   SS30XY = 0.
   SS30XX = 0.
   DO 230 I = F30,L30
     SUM30XY = SUM30XY + (EN(I) * NEWYEN(I))
     SUM30X = SUM30X + NEWYEN(I)
     SUM30XX = SUM30XX + (NEWYEN(I)**2)
     SUM30Y = SUM30Y + EN(I)
230  CONTINUE
   SS30XY = SUM30XY - ((SUM30X*SUM30Y)/D30)
   SS30XX = SUM30XX - ((SUM30X**2)/D30)
   SLOPE30 = SLOPE30 + (SS30XY / SS30XX)
```

```
C
C   NOW COUNT HOW MANY REAL EFFECTS OCCUR IN THE CALULATIONS
C   FOR THE SLOPE OF 30% OF THE DATA.
C
      DO J = 1,F
        DO K = F30,L30
          IF(EN(K) .EQ. EN1(J)) I30 = I30 + 1
        END DO
      END DO
C
C   NOW 55%
C
      I50 = 0
      SUM50XY = 0.
      SUM50X = 0.
      SUM50XX = 0.
      SUM50Y = 0.
      SS50XY = 0.
      SS50XX = 0.
      DO I = F50,L50
        S[illegible]0XY = SUM50XY + (EN(I) * NEWYEN(I))
        [illegible] = SUM50X + NEWYEN(I)
        [illegible] = SUM50XX + (NEWYEN(I)**2)
        SUM5[illegible] = SUM50Y + EN(I)
      END D[illegible]
      SS50[illegible] SUM50XY - ((SUM50X * SUM50Y) / D50)
      SS50[illegible] SUM50XX - ((SUM50X**2) / D50)
      SLOPE50 = SLOPE50 + (SS50XY / SS50XX)
      D[illegible] = 1,F
        [illegible] K = F50,L50
          IF(EN(K) .EQ. EN1(J)) I50 = I50 + 1
        END DO
      END DO
C
C   NOW 68%
C
      I70 = 0
      SUM70XY = 0.

      SUM70X = 0.
      SUM70XX = 0.
      SUM70Y = 0.
      SS70XY = 0.
      SS70XX = 0.
      DO I = F70,L70
        SUM70XY = SUM70XY + (EN(I) * NEWYEN(I))
        SUM70X = SUM70X + NEWYEN(I)
        SUM70XX = SUM70XX + (NEWYEN(I)**2)
        SUM70Y = SUM70Y + EN(I)
      END DO
      SS70XY = SUM70XY - ((SUM70X * SUM70Y) / D70)
      SS70XX = SUM70XX - ((SUM70X**2) / D70)
      SLOPE70 = SLOPE70 + (SS70XY / SS70XX)
      DO J = 1,F
        DO K = F70,L70
          IF (EN(K) .EQ. EN1(J)) I70 = I70 + 1
        END DO
      END DO
C
      DO K = 1,F
        DO I = 1,NFEFF
          IF(EN(I) .EQ. EN1(K)) THEN
              COUNT(I) = COUNT(I) + 1
              TEST(I) = .TRUE.
          END IF
        END DO
      END DO
      DO I=1,NFEFF
              IF (YEN(I) .EQ. 0.0000) THEN
                  YEN(I) = 0.00001
              END IF
         COL1(I) = EN(I) / YEN(I)
         COL2(I) = COL1(I) / NEWD
         ZMDZE(I)= COL2(I)
         NEWE(I) = EN(I) / NEWD
         ZMDZN(I)  = NEWE(I) / NEWYEN(I)
      END DO
C
C   THE IF BLOCK THAT FOLLOWS IS ONLY EXECUTED EVERY
C   ONE HUNDRED TIMES.  A COMMENT OUT TO THE SIDE WILL
C   TELL WHERE THE BLOCK STARTS AND STOPS.
C
      IF (KK .EQ. ITEMP) THEN                ! START OF LOOP
C
C   FIND MEDIAN SIGMA FOR FIRST 11 BIASES
C
        CALL SORT(NEWE,11)
        DO I=1,11
          ZMDZE1(I) = NEWE(I) / YEN(I)
        END DO
        CALL SORT(ZMDZE1,11)
        MEDIANSIGMA = ZMDZE1(6)
```

```
C
C     FIND SLOPE BASED ON 30% ABOUT THE CENTER
C
          MNOZ30XY=0.
          MNOZ30X=0.
          MNOZ30XX=0.
          MNOZ30Y=0.
          MNCZ30XY=0.
          MNCZ30X=0.
          MNCZ30XX=0.
          MNCZ30Y=0.
          MEOZ30XY=0.
          MEOZ30X=0.
          MEOZ30XX=0.
          MEOZ30Y=0.
          MECZ30XY=0.
          MECZ30X=0.
          MECZ30XX=0.
          MECZ30Y=0.
          DO J=F30,L30                 !**MID OF N
            MNOZ30XY =MNOZ30XY + (NEWE(J)  * NEWYEN(J))
            MNOZ30X  =MNOZ30X   + NEWYEN(J)
            MNOZ30XX =MNOZ30XX  + (NEWYEN(J)**2)
            MNOZ30Y  =MNOZ30Y   + NEWE(J)
            MNCZ30XY =MNCZ30XY  + (NEWE(J)  * YENA(J))
            MNCZ30X  =MNCZ30X   + YENA(J)
            MNCZ30XX =MNCZ30XX  + (YENA(J)**2)
            MNCZ30Y  =MNCZ30Y   + NEWE(J)
          END DO
          DO J=F30E,J30E               !**MID OF E
            MECZ30XY =MECZ30XY  + (NEWE(J)  * YENA(J))
            MECZ30X  =MECZ30X   + YENA(J)
            MECZ30XX =MECZ30XX  + (YENA(J)**2)
            MECZ30Y  =MECZ30Y   + NEWE(J)
            MEOZ30XY =MEOZ30XY  + (NEWE(J)  * NEWYEN(J))
            MEOZ30X  =MEOZ30X   + NEWYEN(J)
            MEOZ30XX =MEOZ30XX  + (NEWYEN(J)**2)
            MEOZ30Y  =MEOZ30Y   + NEWE(J)
          END DO
          SSMNOZ30XY=0.
          SSMNOZ30XX=0.
          SSMNCZ30XY=0.
          SSMNCZ30XX=0.
          SSMEOZ30XY=0.
          SSMEOZ30XX=0.
          SSMECZ30XY=0.
          SSMECZ30XX=0.
          SSMNOZ30XY = MNOZ30XY - ((MNOZ30X * MNOZ30Y)/D30)
          SSMNCZ30XY = MNCZ30XY - ((MNCZ30X * MNCZ30Y)/D30)
          SSMNOZ30XX = MNOZ30XX - ((MNOZ30X**2)/D30)
          SSMNCZ30XX = MNCZ30XX - ((MNCZ30X**2)/D30)
          SSMECZ30XY  = MECZ30XY  - ((MECZ30X * MECZ30Y)/D30)
          SSMECZ30XX  = MECZ30XX  - ((MECZ30X**2)/D30)

          SSMEOZ30XY  = MEOZ30XY  - ((MEOZ30X * MEOZ30Y)/D30)
          SSMEOZ30XX  = MEOZ30XX  - ((MEOZ30X**2)/D30)
          MNOZ30=0.
          MNCZ30=0.
          MEOZ30=0.
          MECZ30=0.
          MNOZ30 = (SSMNOZ30XY / SSMNOZ30XX)
          MNCZ30  = (SSMNCZ30XY  / SSMNCZ30XX)
          MEOZ30  = (SSMEOZ30XY  / SSMEOZ30XX)
          MECZ30 = (SSMECZ30XY / SSMECZ30XX)
          MNOZ50XY=0.
          MNOZ50X=0.
          MNOZ50XX=0.
          MNOZ50Y=0.
          MNCZ50XY=0.
          MNCZ50X=0.
          MNCZ50XX=0.
          MNCZ50Y=0.
          MEOZ50XY=0.
          MEOZ50X=0.
          MEOZ50XX=0.
          MEOZ50Y=0.
          MECZ50XY=0.
          MECZ50X=0.
          MECZ50XX=0.
          MECZ50Y=0.
```

```
C
C    FIND SLOPE BASED ON 50% OF THE CENTER
C
        DO K=F50,L50          !**MID OF N
          MNOZ50XY= MNOZ50XY + (NEWE(K) * NEWYEN(K))
          MNOZ50X  = MNOZ50X + NEWYEN(K)
          MNOZ50XX = MNOZ50XX + (NEWYEN(K)**2)
          MNOZ50Y  = MNOZ50Y + NEWE(K)
          MNCZ50XY= MNCZ50XY + (NEWE(K) * YENA(K))
          MNCZ50X = MNCZ50X + YENA(K)
          MNCZ50XX= MNCZ50XX + (YENA(K)**2)
          MNCZ50Y = MNCZ50Y + NEWE(K)
        END DO
        DO K=F50E,J50E         !**MID OF E
          MECZ50XY = MECZ50XY + (NEWE(K) * YENA(K))
          MECZ50X  = MECZ50X + YENA(K)
          MECZ50XX = MECZ50XX  + (YENA(K)**2)
          MECZ50Y  = MECZ50Y + NEWE(K)
          MEOZ50XY= MEOZ50XY + (NEWE(K) * NEWYEN(K))
          MEOZ50X = MEOZ50X + NEWYEN(K)
          MEOZ50XX= MEOZ50XX + (NEWYEN(K)**2)
          MEOZ50Y = MEOZ50Y + NEWE(K)
        END DO
        SSMNOZ50XY=0.
        SSMNOZ50XX=0.
        SSMNCZ50XY=0.
        SSMNCZ50XX=0.

        SSMEOZ50XY=0.
        SSMEOZ50XX=0.
        SSMECZ50XY=0.
        SSMECZ50XX=0.
        SSMNOZ50XY = MNOZ50XY - ((MNOZ50X * MNOZ50Y)/D50)
        SSMECZ50XY = MECZ50XY - ((MECZ50X * MECZ50Y)/D50)
        SSMNOZ50XX = MNOZ50XX - ((MNOZ50X**2)/D50)
        SSMNCZ50XX = MNCZ50XX - ((MNCZ50X**2)/D50)
        SSMNCZ50XY= MNCZ50XY- ((MNCZ50X * MNCZ50Y)/D50)
        SSMECZ50XX= MECZ50XX- ((MECZ50X**2)/D50)
        SSMEOZ50XY= MEOZ50XY- ((MEOZ50X * MEOZ50Y)/D50)
        SSMEOZ50XX= MEOZ50XX- ((MEOZ50X**2)/D50)
        MNOZ50=0.
        MNCZ50=0.
        MEOZ50=0.
        MECZ50=0.
        MNOZ50 = (SSMNOZ50XY / SSMNOZ50XX)    !*SLOPE MID N OZ
        MECZ50 = (SSMECZ50XY / SSMECZ50XX)    !*SLOPE MID E CZ
        MNCZ50  = (SSMNCZ50XY  / SSMNCZ50XX)     !*SLOPE MID N CZ
        MEOZ50  = (SSMEOZ50XY  / SSMEOZ50XX)     !SLOPE  MID E OZ
        OPEN(UNIT=9,TYPE='NEW',FILE='SIMON.DAT')
        WRITE(9,793)KK,NFEFF,F,POSEFF
        WRITE(9,794)MAG(1)
        WRITE(9,798)
793          FORMAT(1X,'RUN',I5,3X,'K=',I5,3X,' REALS=',I5,3X,
     X'  + EFFECTS=',I5)
794     FORMAT(1X,'MAGNITUDE OF EFFECT IS',F7.4//)
        DO I=1,NFEFF
             IF(TEST(I))then
           WRITE(9,791)EN(I),NEWE(I),ZMDZE(I),ZMDZN(I)
             ELSE
           WRITE(9,792)EN(I),NEWE(I),ZMDZE(I),ZMDZN(I)
             END IF
        END DO
        WRITE(9,795)MEDIANSIGMA
        WRITE(9,850)MNOZ30
        WRITE(9,851)MNCZ30
        WRITE(9,852)MEOZ30
        WRITE(9,853)MECZ30
        WRITE(9,854)MNOZ50
        WRITE(9,855)MNCZ50
        WRITE(9,856)MEOZ50
        WRITE(9,857)MECZ50
C
C    WRITE SLOPE SCORES OUT FOR FREQ. DIST.
C
                  WRITE(11,*)MEDIANSIGMA,MECZ30,MECZ50
C
C    WRITEOUT DATA FOR BMDP
C
        DO I=1,NFEFF
          MAT(I,NEWK) = NEWE(I)
        END DO
```

```
C
 795      FORMAT('  MED. OF ZMD/ZE',F15.5)
 850      FORMAT('  MID. OF N OZ30',F15.5)
 851      FORMAT('  MID. OF N CZ30',F15.5)
 852      FORMAT('  MID. OF E OZ30',F15.5)
 853      FORMAT('  MID. OF E CZ30',F15.5)
 854      FORMAT('  MID. OF N OZ50',F15.5)
 855      FORMAT('  MID. OF N CZ50',F15.5)
 856      FORMAT('  MID. OF E OZ50',F15.5)
 857      FORMAT('  MID. OF E CZ50',F15.5)
 790      FORMAT('             MD                    ZMD              ZMD/ZE
     *        ZMD/ZN')
C         CLOSE(UNIT=9)
          ITEMP = ITEMP+100
          NEWK = NEWK+1
        END IF                                  ! END OF LOOP
C
 791    FORMAT(F15.5,'R',3F15.5)
C
C    SUM UP FOR MEANS AND VARIANCES
C
        DO 45 I=1,NFEFF
          E(I)=E(I)+EN(I)
          E1(I) = E1(I) + EN1(I)
          ESS(I)=ESS(I)+EN(I)**2
 45         CONTINUE
        T30 = T30 + I30
        T50 = T50 + I50
        T70 = T70 + I70
C
C    INC. COUNTER FOR THE 50 COLUMNS IN MATRIX MAT
C
 100 CONTINUE
C
C    NOW WRITE MAT OUT FOR BMDP ANALYSIS
C
     OPEN(UNIT=15,TYPE='NEW',NAME='SIMONBMD.DAT',RECORDSIZE=800)
     DO J=1,NFEFF
       WRITE(15,792)(MAT(J,I),I=1,50)
     END DO
     CLOSE(UNIT=15)
C
C    GET MEANS AND VARIANCES
C
     REP=NR
     DO 120 J=1,NFEFF
       E(J)=E(J)/REP
       E1(J) = E1(J) / REP
       NEWE(J) = E(J) / NEWD
       ZMDZN(J) = NEWE(J) / NEWYEN(J)
       IF(YEN(J) .EQ. 0.) YEN(J) = .00001
       COL1(J) = E(J) / YEN(J)
       COL2(J) = COL1(J) / NEWD

      ESS(J)=ESS(J)/REP-E(J)**2
      ESS(J)= SQRT(ESS(J))/NEWD
 120      CONTINUE
     SLOPE30 = (SLOPE30 / REP)/ NEWD
     SLOPE50 = (SLOPE50 /REP) / NEWD
     SLOPE70 = (SLOPE70 / REP)/ NEWD
C
     OPEN(UNIT=6,TYPE='NEW',FILE='WES.DAT')
     OPEN(UNIT=7,TYPE='NEW',FILE='ZMD.DAT')
C
     WRITE(6,753) NSCELL
 753         FORMAT('           NUMBER OF SUBJECTS PER CELL IS',I3)
     WRITE(6,755) NR
 755         FORMAT('           NUMBER OF RUNS IS',I6)
     WRITE(6,780) F
 780         FORMAT('           NO. REAL EFFECTS IN THIS DESIGN IS',I5)
     WRITE(6,785)(MAG(J),J=1,F)
     WRITE(6,789)
     WRITE(6,790)
     DO 130 J=1,NFEFF
 130           WRITE(6,550) J,E(J),COUNT(J),NEWE(J),YEN(J),
      X     EFF(J),COL1(J),COL2(J),ZMDZN(J),ESS(J),YEN2(J)
C
 785 FORMAT(10X,'MAGNITUDE OF THE EFFECTS ARE ',8F7.4)
 790 FORMAT(10X,'____________________________________
      X_________________________')
 789 FORMAT(10X,'     M.D.     EFFECTS        ZMD           ZE
      X       MD/ZE     ZMD/ZE     ZMD/ZN        STD. sd.   E.N. sd.')
 550         FORMAT(10X,I3,F8.3,1X,I7,2X,F8.3,2X,F8.3,1X,
      X'(',I2,')',3F10.3,2F10.3)
 551         FORMAT(10X,I3,F8.3,1X,F8.3,1X,'(',I2,')',2F10.3)
 792         FORMAT(50F15.5)
     DO K=1,NFEFF
       WRITE(7,*)NEWE(K)
     END DO
     CLOSE(UNIT=7)
C
     STOP
     END
C
C
```

```
C
      SUBROUTINE SORT(X,N)
      DIMENSION X(127)
      M=N
    1 K=0
      DO 2 I=2,M
        J=I-1
        IF(X(J) .LE. X(I)) GO TO 2
        K=J
        S=X(I)
        X(I)=X(J)
        X(J)=S
    2 CONTINUE
      IF(K .EQ. 0)RETURN
      M=K
      GO TO 1
      END
C
C
C
C
C
C
      SUBROUTINE RNORM01(N,X,SEED)
C
C     TO GENERATE RANDOM NORMAL NUMBERS FROM UN. RANDOM
C     NUMBERS BY BOX AND MULLER(1958) METHOD. IF R1 AND R2 ARE
C     2 UN. RAND NUMBERS, THEN
C             Z1=SQRT(-2.*LOG(R1))*COS(2*PI*R2)
C             Z2=SQRT(-2.*LOG(R1))*SIN(2*PI*R2)
C     ARE A PAIR FORM N(0,1).
C
      IMPLICIT REAL*4 (M)
      DIMENSION X(128)
      DO 50 I=1,N,2
        R1=MTH$RANDOM(SEED)
        R2=MTH$RANDOM(SEED)
        X(I)=SQRT(-2.*LOG(R1))*COS(6.2831853*R2)
        X(I+1)=SQRT(-2.*LOG(R1))*SIN(6.2831853*R2)
   50 CONTINUE
      RETURN
      END
C
C
C
C


C
C
      SUBROUTINE SCOR(J,N,SCORE,VAR)
C
C     COMPUTES THE E(x) OF THE JTH OF N RANKED NORMAL
C     SCORES.
C        SCORE RETURNS WITH THE EXPECTED NORMAL SCORE.
C        VAR RETURNS WITH THE VARIANCE OF THE NORMAL SCORE.
C     COMPUTE THE CONTENT OF THE INTEGRAL LOG FORM
      C = FACL(N)-FACL(J-1)-FACL(N-J)
C
C     GET THE APPROX. NORMAL SCORE
C
      AN = SCR(J,N)
      ANN = AN
    3 ANN = ANN-.1
      IF (FUN(ANN,C,J,N) .GT. 0.) GO TO 3
      RBOT = ANN
    4 ANN = ANN+.1
      IF (FUN(ANN,C,J,N) .GT. 0.) GO TO 4
      RTOP = ANN
      RNG = AN-RBOT
      XU = AN
      FU = XU*FUN(XU,C,J,N)
      W = RNG/100.
      PRT1 = 0.
      PRT1V = 0.
```

```
C
C
C     START THE FIRST SIMPSON INTEGRATION
C
C
10    XL = XU-W
      XM = (XU+XL)/2.
      FM = XM*FUN(XM,C,J,N)
      FL = XL*FUN(XL,C,J,N)
      P1 = PRT1+(FL+4.*FM+FU)*W/6.
      PRT1V = PRT1V+(XL*FL+4.*XM*FM+XU*FU)*W/6.
      IF (PRT1 .EQ. P1) GO TO 15
      W = W*1.1
      XU = XL
      FU = FL
      PRT1 = P1
      GO TO 10
15    CONTINUE
      RNG = RTOP - AN
      XL = AN
      FL = XL*FUN(XL,C,J,N)
      W = RNG/100.
      PRT2 = 0.
C
C


C
      PRT2V = 0.
C
C     START SECOND SIMPSON INTEGRATION
C
20    XU = XL+W
      XM = (XU+XL)/2.
      FM = XM*FUN(XM,C,J,N)
      FU = XU*FUN(XU,C,J,N)
      P2 = PRT2+(FU+4.*FM+FL)*W/6.
      PRT2V = PRT2V+(XU*FU+4.*XM*FM+XL*FL)*W/6.
      IF (PRT2 .EQ.P2) GO TO 25
      W = W*1.1
      XL = XU
      FL = FU
      PRT2 = P2
      GO TO 20
25    CONTINUE
      SCORE = PRT1+PRT2
      SCR2 = PRT1V+PRT2V
      VAR = SCR2-SCORE*SCORE
      RETURN
      END
C
C
C
C
      FUNCTION FUN(X,C,J,N)
C
C     COMPUTES VALUE OF FUNCTION TO BE INTEGRATED, FIRST IN LOG
C     FORM, THEN IN DECIMAL FORM IF LARGE ENOUGH.
C     C = N!/(J-1)!/(N-J)! - THE CONSTANT OF INTEGRATION
C     FUN = C*P**(N-J)*Q**(J-1)*Z
C     WHERE P IS THE PROPORTION BELOW  Q IS THE PROPORTION ABOVE
C     X ON THE NORMAL CURVE; AND Z IS THE ORDINATE OF THE NORMAL
C     CURVE AT X.
C
      DOUBLE PRECISION X,POFZ,P,Q,XJ,XN,FUN,FL
      XJ = J
      XN = N
      P = POFZ(X)
      Q = 1.D0-P
      FUN = 0.D0
      IF (P .LE. 0.D0 .OR. Q .LE. 0.D0)RETURN
      FL = C+DLOG(P)*(XN-XJ)+DLOG(Q)*(XJ-1.D0)-X*X/2.D0-0.918938533D0
      IF (FL .LT. -40.)RETURN
      FUN = DEXP(FL)
      RETURN
      END
C
C
```

```
C
      FUNCTION FACL(K)
C
C     COMPUTES THE NATURAL LOG OF K!
C     ABOVE 36 USES STIRLINGS APPROXIMATION
C
      IF (K .GT. 36) GO TO 20
      FACL = 0.
      IF (K .LE. 1)RETURN
      DO 10 I=2,K
        X = I
10    FACL = FACL+ALOG(X)
      RETURN
20    X = K
      FACL = .918938533+ALOG(X)*(X+.5)-X+1./(12.*X)
      RETURN
      END
C
C
      FUNCTION SCR(J,N)
C
C     RAPID APPROXIMATION TO THE JTH OF N EXPECTED NORMAL SCORES.
C     USES BLOMS ALGORITHM WITH CORRECTIONS PROPOSED BY
C     HARTLEY, BIOMETRIKA, 1961, 48, 151-165.
C
      XJ = J
      XN = N
      X = ALOG10(XN)
      IF (N .GT. 400) GO TO 10
      A1 = .315065+.057974*X-.009776*X*X
      A2 = .327511+.058212*X-.007909*X*X
      GO TO 20
10    A1 = .3752+.00976*X+.00008*X*X
      A2 = .3866+.01418*X+.00029*X*X
20    A = A2
      IF (J .EQ.1.D0 .OR. J .EQ. N) A=A1
      SCR = ZINV((XJ-A)/(XN-2.*A+1.))
      SCR = -SCR
      RETURN
      END
C


C
      FUNCTION ZINV(P)
C
C     QUICK APPROXIMATION TO THE INVERSE OF THE NORMAL DIST. FUNCTION
C     EMERSON, BRMI, 1979,11, 397-398.
C
      SGN = -1.
      Q = P
      IF (Q-.5)2,2,1
1     SGN = 1.
      Q = 1.-Q
2     Z = SQRT(-2.*ALOG(Q))
      W = Z*(.18926+Z*.001308)
      W = 1.+Z*(1.432788+W)
      Q = .802853+Z*.010328
      W = (2.515517+Z*Q)/W
      ZINV = (Z-W)*SGN
      RETURN
      END
C
C
      DOUBLE PRECISION FUNCTION POFZ(XX)
C
C     NORMAL PROBABILITY INTEGRAL, -INFINITY TO X; X>0
C     ALGRORITHM 26.2.11 P.932 ZELEN _SEVERO
C
      DOUBLE PRECISION X,XX,S,T,C,XN,SN
      X= DABS(XX)
      POFZ = 1.D0
      IF (X .GT. 8.35) GO TO 20
      S = X
      T = X
      C = X*X
      XN = 0.D0
5     XN = XN+1.D0
      T = T*C/(2.D0*XN+1.D0)
      SN = S+T
      IF (SN .EQ. S) GO TO 10
      S = SN
      GO TO 5
10    POFZ = .5D0+.398942280401432700*DEXP(-C/2.D0)*S
20    IF (XX .LT. 0.D0) POFZ=1.D0-POFZ
      RETURN
      END
```

APPENDIX D

COMPUTER PROGRAM FOR GENERATING EXPECTED VALUES OF NORMAL AND HALF-NORMAL ORDER STATISTICS

David G. Weinman, Ph.D.

This program was written in VAX-BASIC and run on the Hollins VAX 11-780 computer. The following program is for generating <u>half-normal</u> order statistics. To generate normal order statistics, delete the two references to absolute values (ABS) in line 230 and change comments regarding "half-normal" to "normal."

```
100 Rem -- Generate half-normal order statistics
110 DECLARE INTEGER GAP, I, J, N, NREP, NXT, P, REP, SORTFLAG, T, X
    DECLARE REAL A(128), ASUM(128), ADD, N1, TEMP, MEAN, STD
    RANDOMIZE
    NREP = 5000
120 FOR N = 20 TO 31
130      N1 = N/2
140      IF INT(N1) = N1 THEN
            K1 = N1
         ELSE
            K1 = N1 + 1
         END IF
         Z$ = STR$(N)
         DATA$ = "FACULTY:[DAVE.SIMON]HALFN" + Z$ + ".DAT"
150      FOR I = 1 TO N
            ASUM(I) = 0
         NEXT I
160      FOR REP = 1 TO NREP
            GOSUB 200  ! Generate N half-normal(0, 1) values
            GOSUB 400  ! Sort the values
            FOR I = 1 TO N
               ASUM(I) = ASUM(I) + A(I)
            NEXT I
170      NEXT REP
         GOSUB 600  ! Calculate mean of order statistics and print
180 NEXT N
    GO TO Finish
190 !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
```

```
200 ! Generate N half-normal variates (Polar Marsaglia method)
210 FOR I = 1 TO K1
220      V1 = 2 * RND - 1
         V2 = 2 * RND - 1
         R2 = V1*V1 + V2*V2
         IF R2 > 1 THEN 220
230      Y = SQR( (-2 * LOG(R2) ) / R2)
         A(2*I-1) = ABS(V1 * Y)
         A(2*I)   = ABS(V2 * Y)
240 NEXT I
250 RETURN
300 !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
400 ! Subroutine: Sort the array A
410 GAP = N
420 WHILE GAP > 1
430    GAP = GAP / 2
440 ! Put numbers GAP positions apart in order.
       TOP = N - GAP
       SORTFLAG = 0
       WHILE SORTFLAG = 0
          SORTFLAG = 1
          FOR I = 1 TO TOP
              NXT = I + GAP
              IF A(I) > A(NXT) THEN
                 SORTFLAG = 0
                 TEMP    = A(I)
                 A(I)    = A(NXT)
                 A(NXT)  = TEMP
              END IF
          NEXT I
       NEXT
    NEXT


450 RETURN
500 !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
600 ! Calculate mean of order statistics
610 OPEN DATA$ FOR OUTPUT AS FILE #1
620 FOR I = 1 TO N
        MEAN = ASUM(I) / NREP
        PRINT #1, MEAN
630 NEXT I
640 CLOSE #1
680 RETURN
690 !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
700 Finish: END
```

APPENDIX E

DEVIATION OF STANDARD ERROR OF THE CONTRASTS

The standard error of the contrasts (s_c) is the same as the standard error of the mean differences. The general equation for this is:

$$s_C = \sqrt{\frac{\sigma_1^2}{n/2} + \frac{\sigma_2^2}{n/2}} \qquad [\sigma_1 = \sigma_2]$$

$$s_C = \sqrt{\frac{2\sigma^2}{n/2}}$$

$$s_C = \sqrt{\sigma^2 \frac{2}{n/2}}$$

$$s_C = \sqrt{\sigma^2 \frac{4}{n}}$$

APPENDIX F

PREDICTING R-SPILLOVER USING CONTRASTS AND CONTRAST STANDARD DEVIATIONS

David G. Weinman, Ph.D.

[The author asked Dr. Weinman to investigate whether or not the contrast standard deviation at each rank (see Table 1 for example) could be used to estimate the R-spillover. The R-spillover, it was felt, could be useful to an investigator who wants to decide whether it is likely that a smaller contrast may occur at a rank below where other techniques had determined the contrast was likely to be a real one. In the discussion below, Dr. Weinman explains what might be done, how complex it can be to do it correctly, and the theoretical limits of this approach.]

For any sample size $n \geq 2$ from a normal population, the longest and smallest order statistics do not have a normal distribution. However, we attempt here to estimate spillover by using an assumption of normality, because a normal distribution often gives reasonable (approximately correct) results.

Consider the situation of $n = 31$ effects, and suppose we add 1.67σ to eight of these effects. Our 1.67σ is equal to $[1.67 \div \sqrt{4/32}]s_c$, where s_c is the standard error of a contrast, so we actually add $[1.67 \div \sqrt{4/32}]$, or 4.723 to each of eight contrasts.

Then we have 31 effects, of which 23 are "error" effects, and 8 are "real" effects. The 23 error effects are distributed as 23 order statistics (from a sample of 23) from a normal (0, 1) distribution, while the 8 real effects are distributed as the order statistics from a sample of 8 from a normal $[(1.67 \div \sqrt{4/32}), 1.00]]$ distribution.

From our simulation, the means and standard deviations are as follows for the longest 3 of 23 order statistics and the smallest 3 of 8 order statistics, with 4.723 added to the three smallest. We use x for the error effects and y for the real effects.

With some 5000 replications, we would expect to find 7.0, 21.5, and 174.0 real effects in positions 21, 22, and 23 respectively. On one run we found 11, 43, and 186 real effects in those positions. To summarize.

Position	Expected Number of Real Effects	Actual Number of Real Effects
21	7.0	11
22	21.5	43
23	174.0	186

The results for positions 21 and 23 are encouraging, but the prediction at position 22 is off by 100%.

This case -- 8 real effects of size 1.67σ -- is one of the cleanest situations to deal with, because 1.67σ is a large effect. With smaller real effects, many more probabilities must be calculated and the expected number of real effects is a poorer estimate of the actual number of real effects found in simulations.

Order	Contrast	Contrast Std. Deviation	Order	Contrast	Contrast Std. Deviation
x_{21}	1.21934	0.33574	y_1	3.29987	0.61065
x_{22}	1.48096	0.27804	y_2	3.87125	0.48929
x_{23}	1.91716	0.48899	y_3	4.25065	0.44870

The spillover rate from real effects into error positions then involves the probabilities $P(y_i < x_j)$ that some y_i is less than some x_j. Dropping the subscripts i and j, we have

$$P(y < x) = P(y-x < 0)$$

$$= P\left(z < \frac{\mu_x - \mu_y}{\sigma_x^2 + \sigma_y^2}\right.$$

where z is a standard N[0,1].

For i = 1,2 and j = 22,23, these probabilities are:

$P(y_1 < x_{23}) = .0384$
$P(y_1 < x_{22}) = .0057$
$P(y_1 < x_{21}) = .0014$
$P(y_2 < x_{23}) = .0024$
$P(y_2 < x_{22}) = .0000$

Then, assuming independence, we have the following probabilities of real effects spilling over into positions 21, 22, and 23.

Position	P (Real effect in position)
21	.0014
22	.0043 (= .0057 - .0014)
23	.0348 (= .0384 - .0057 + .0024)

APPENDIX G

PROGRAM FOR GENERATING GUARDRAILS

David G. Weinman, Ph.D.

This program was written in VAX-BASIC and run on the Hollins VAX 11-780 computer. The following program is for generating guardrails for half-normal plots. To generate guardrails for normal order statistics, delete the two references to absolute values (ABS) in line 250 and change comments regarding "half-normal" to "normal."

```
100 ! PROGRAM GUARD -- Obtain guardrails for half-normal plots
    !  Generate half-normal variables
    !  Find percentiles of largest order statistic / slope estimate
110 DECLARE INTEGER GAP, I, J, K1, M, N, NREP, NXT, P, REP, SORTFLAG
    DECLARE REAL NORM(64), A(64), LARGE(10000), R2, V1, V2, Y
    DECLARE REAL A05, A10, A20, A40, B, N1, R, SSX, SXY, TEMP
    RANDOMIZE

120 NREP   = 5000
    ! Open output file and print header
    OUTFILE$ = "FACULTY:[DAVE.SIMON]ALPHA31.OUT"
    OPEN OUTFILE$ FOR OUTPUT AS FILE #1

130 PRINT #1, "Half Normal"
    PRINT #1, "Number of replications        "; NREP
    PRINT #1
    PRINT #1, "                              Values of alpha"
    PRINT #1, "  #          .05       .10       .20       .40"
    PRINT #1, "----------------------------------------------"
    A$       = " ##         ##.###    ##.###    ##.###    ##.###"

    ! Main program: N is the number of effects being considered
    ! Polar Marsaglia method generates an even number of values,
    ! so determine K1 and generate 2K1 values.
    ! Open input file and read half-normal values.
140 FOR N = 31 TO 20 STEP -1

        N1 = N / 2
        IF INT(N1) = N1 THEN
           K1 = N1
        ELSE
           K1 = N1 + 1
        END IF

        INFILE$  = "FACULTY:[DAVE.SIMON]HALFN" + STR$(N) + ".DAT"
        OPEN INFILE$ FOR INPUT AS FILE #2
        FOR I = 1 TO N
           INPUT #2, NORM(I)
        NEXT I
        CLOSE #2
```

```
150      FOR REP = 1 TO NREP
             GOSUB 200  ! Generate N half-normal(0, 1 ) values
             GOSUB 400  ! Sort array A and save largest value
             GOSUB 500  ! Calculate slope and divide largest value by it
160      NEXT REP
         GOSUB 600 ! Sort the values of LARGE
         GOSUB 800 ! Print results
170 NEXT N
    CLOSE #1
180 GO TO Finish
190 !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
200 ! Generate N half-normal variates (Polar Marsaglia method)
    ! as given in Morgan: Elements of Simulation
230 FOR I = 1 TO K1
240      V1 = 2 * RND - 1
         V2 = 2 * RND - 1
         R2 = V1*V1 + V2*V2
         IF R2 > 1 THEN 240
250      Y = SQR( (-2 * LOG(R2) ) / R2)


         A(2*I-1) = ABS(V1 * Y)
         A(2*I)   = ABS(V2 * Y)
260 NEXT I
270 RETURN
390 !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
400 ! Subroutine: Sort th` array A (Shellsort)
410 GAP = N
420 WHILE GAP > 1
430     GAP = GAP / 2
440 ! Put numbers GAP positions apart in order.
        TOP = N - GAP
        SORTFLAG = 0
        WHILE SORTFLAG = 0
           SORTFLAG = 1
           FOR I = 1 TO TOP
               NXT = I + GAP
               IF A(I) > A(NXT) THEN
                  SORTFLAG = 0
                  TEMP     = A(I)
                  A(I)     = A(NXT)
                  A(NXT)   = TEMP
               END IF
            NEXT I
        NEXT
    NEXT
    LARGE(REP) = A(N)
450 RETURN
500 !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
510 ! Subroutine: Calculate slope B and divide LARGE by it.
520 SXX = 0.0
530 SXY = 0.0
540 FOR I = 1 TO N
        SXX = SXX + NORM(I)^2
        SXY = SXY + NORM(I) * A(I)
    NEXT I
550 B = SXY / SXX
560 LARGE(REP) = LARGE(REP) / B
580 RETURN
590 !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
```

```
600 ! Subroutine: Sort the first 2/5 values of array LARGE
610    FOR I = 1  TO 1 + 2 * NREP / 5
           LARG = LARGE(I)
           PSTN = I
           FOR NXT = I + 1 TO NREP
               IF LARGE(NXT) > LARG THEN
                  LARG = LARGE(NXT)
                  PSTN = NXT
               END IF
           NEXT NXT
           LARGE(PSTN) = LARGE(I)
           LARGE(I)    = LARG
620    NEXT I
650 RETURN
690 !!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
800 ! Subroutine: Calculate alpha values and print results
820 A05 = LARGE(NREP / 20) + LARGE(1 + NREP / 20)
    A05 = A05 / 2
    A10 = LARGE(NREP / 10) + LARGE(1 + NREP / 10)
    A10 = A10 / 2
    A20 = LARGE(NREP / 5) + LARGE(1 + NREP / 5)

    A20 = A20 / 2
    A40 = LARGE(2 * NREP / 5) + LARGE(1 + 2 * NREP / 5)
    A40 = A40 / 2
    PRINT #1 USING A$; N, A05, A10, A20, A40
830 RETURN
999 Finish: END
```

NOTATIONS AND TERMINOLOGY

α	Probability of rejecting the null hypothesis when it is true; probability of making a Type 1 error.
Contrast	Difference between two means (the high and low, or positive and negative conditions of a balanced, two-level factorial design; an effect.
d_r	Size of r real effects.
e	Number of error contrasts (or null contrasts) in the results of a 2^f or $2^{(f-F)}$ experiment.
Effect	Size of the mean difference or contrast.
e.v.n.o.s.	Expected values for normal order statistics.
E-spillover	Error contrasts that fall at ranks where real effects were introduced prior to adding an error component.
Expected value for normal order statistics	Positions on the normal probability scale for each of n items in rank order.
Half-normal Plot	Plot of absolute contrasts on half-normal probability paper.
i	Rank position of a value in a set of ordered values.
Intended error ranks	Ranks where no real increments were added before real and error components were combined and reranked.

NOTATIONS AND TERMINOLOGY (cont'd)

k	Number of experimental effects. In unreplicated 2^f designs, $k = (n - 1)$.
n	Number of independent observations in an experiment. In an unreplicated 2^f factorial design, $n = k+1$. Also $n = (r + e)$.
$\underline{N}[0,1]$	Normally distributed population with a mean of zero and a variance of one.
Normal plot	Ordered signed contrasts plotted on normal probability paper.
PER	Probability of a non-zero "family" error rate, dependent upon number of contrasts being tested.
P_i	Normal probability scale value at rank i.
P'_i	Half-normal probability scale value at rank i.
r	Number of real or non-null effects in the set of k contrasts.
$r+$	Number of positive real effects (r- is number of negative real effects).
$\#R_{i,k}$	Number of real effects that occurred at rank i of the ordered set of k contrasts during a Monte Carlo simulation (of 5000 runs in this report).
$\%R_{i,k}$	Percent real effects falling at rank i for k effects.

R-spillover	Real effects that fall at ranks intended to be error contrasts.
σ	Population sigma or standard deviation. σ^2 equals the population variance, sometimes referred to as the error variance.
s	Estimation of the population σ
s_f	Final estimation of the population σ
s_c	Standard error of the contrasts, obta:·.ed by multiplying the estimated population sigma by the square root of four over n.
$s_{i,k}$	Standard deviation of a contrast at rank i of k contrasts.
t_i^{Δ}	Test statistic proposed by Daniel used to determine the probability that an effect of a certain size will occur. Determines location of guardrails.
SL	Slope of the e contrasts on a normal or a half-normal plot. May be calculated by some least squares regression technique. The slope of standardized contrasts serves is used to estimate the population sigma.
2^f or $2^{(f-p)}$	Factorial or fractional factorial experiment with f factors at two levels each. (f-p) represents a $1/2^p$ fraction of a 2^f factorial. [Note: In two papers in Section VII on "Relevant Papers," the symbols 2^n and 2^n3^m were used as the authors had used them. The n in that case is f as used in this report.]

NOTATIONS AND TERMINOLOGY (cont'd)

$x_{i,k}$ or $x'_{i,k}$	Contrast (i.e., mean difference) at rank i of k ordered contrasts for normal or half-normal data. The half-normal symbol is indicated by the prime.
$X_{i,k}$ or $X'_{i,k}$	Standardized contrasts, obtained by dividing raw contrasts by estimated population sigma for normal and half-normal data.
Versions S, X, R, SR, or XR	Different techniques investigated by Zahn for estimating the population sigma using the slope of the standardized contrasts.
$x_{a,k}$	Best estimate of the standard error of contrasts in a null experiment with k contrasts, with a = integer nearest 0.638k.
$Z_{i,k}$ or $Z'_{i,k}$	Estimated value of order statistics at rank i any set of k values for normal or half-normal data. [Note: The k is used here to represent the number of contrasts being plotted, although most tables of e.v.n.o.s. use the symbol n.]

END

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